Parametric absorption in electron cyclotron resonance heating of tokamak plasmas

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A theoretical study of parametric absorption in electron cyclotron resonance heating (ECRH) of tokamak plasmas is presented. The problem of primary and secondary decays including their temporal and spatial growth rates is studied. In the nonlinear phase of development of the excited waves, a cascading saturation mechanism is invoked for both convective and absolute instabilities. Anomalous absorption frequencies and anomalous absorption lengths are evaluated for a variety of parametric excitations.

I. INTRODUCTION

The excitation of intense high-frequency plasma waves represents a promising method for additional heating of tokamak plasmas. The recent development of gyrotrons (electron-cyclotron masers)[1] has made it possible to carry out electron cyclotron resonance heating (ECRH) in tokamaks.[2] Most papers on plasma heating in this frequency range have dealt with mainly linear theory[3] while, more recently, a few papers[4–7] have considered nonlinear interactions of a driver pump with plasmas.

The electric field intensities generated by current gyrotrons are sufficient for exciting parametric processes in tokamak plasmas.[8] Consequently, nonlinear processes in tokamak plasma heating at ECR could play a significant role. Parametric excitations in the electron cyclotron frequency range (ECFR) have also been observed in other plasmas.[9] Understanding of parametric excitations in a plasma driven with external e.m. power in the ECFR is important not only for heating but also because it presents a possibility of finding new means for probing and controlling of the plasma density and current profiles.

This paper is organized as follows: Parametric excitations and their thresholds are studied in Secs. II and III, and secondary decay processes in the case of absolute and convective instabilities in Sec. IV. In Sec. V we give the nonlinearly absorbed energy through cascade saturation and summarize the anomalous frequencies and absorption lengths for the various parametric excitations. In Sec. VI we outline some of the limitations of cascade saturation mechanism, and in Sec. VII we give the conclusion emphasizing the consequences of parametric processes in contemporary tokamak experiments.

II. PARAMETRIC COUPLING IN THE ECFR

In what follows we shall consider nonlinear interactions of a pump in the form $\mathbf{E}(t) = \mathbf{E}_0 \sin(\omega_0 t - \mathbf{k}_0\cdot\mathbf{r})$ with a low β, strongly magnetized plasma ($\Omega_e > \omega_{pe}$). The linear phase of development of parametric instabilities is described by the nonlinear (with respect to the external electric field), mode–mode coupling dispersion relation obtained through the well-known plasma dielectric permittivity formalism;[10] it can be written as follows:

$$\epsilon^{(0)} + \chi_i^{(0)}(1+\chi_e^{(0)})\left(\frac{e^{(+n)}}{\epsilon^{(+n)}} + \frac{e^{(-n)}}{\epsilon^{(-n)}}\right) = 0, \quad n = 1,2\dots . \qquad (1)$$

Here $\epsilon^{(0)} = \epsilon(\omega,\mathbf{k}) \equiv \epsilon_L$ is the low-frequency plasma mode dielectric permittivity; ω and $\mathbf{k}$ are the frequency and the wave vector of the low-frequency mode; $\epsilon^{(\pm n)} = \epsilon(\omega \pm n\omega_0, \mathbf{k} \pm n\mathbf{k}_0) \equiv \epsilon_H$ is the high-frequency magnetized plasma mode dielectric permittivity; $\omega \pm n\omega_0$, $\mathbf{k} \pm n\mathbf{k}_0$ are the frequencies and the wave vectors of the side bands (Stokes and anti-Stokes); $\chi_i^{(0)} \equiv \chi_i(\omega,\mathbf{k})$ and $\chi_e^{(0)} \equiv \chi_e(\omega,\mathbf{k})$ are the low-frequency ion and electron plasma susceptibilities, respectively. The coefficients $e^{(\pm n)}$ describe the coupling of the upper and lower sidebands with the low-frequency plasma mode through the pump electric field. The coupling coefficients, maximized with the following selection rules: $n\omega_0(\mathbf{k}_0) \simeq \omega_H(\mathbf{k}_H) + \omega(\mathbf{k})$ and $n\mathbf{k}_0 \simeq \mathbf{k}_H + \mathbf{k}$ ($\omega_H \equiv \omega \pm n\omega_0$ and $\mathbf{k}_H \equiv \mathbf{k} \pm n\mathbf{k}_0$), are given by

$$e_{\max}^{(\pm n)} = \frac{|(\mathbf{k} \pm n\mathbf{k}_0)\mathbf{r}_{EB}|^{2n}}{|\mathbf{k} \pm n\mathbf{k}_0|^{2n}} \frac{k^{2n}}{(n!)^2 4^n}, \qquad (2)$$

where ω_0 and $\mathbf{k}_0$ are the angular frequency and the wave vector of the pump; and $\mathbf{r}_{EB}$ is the vector of electron oscillation amplitude in the electric field of the pump and confining (toroidal) magnetic field $\mathbf{B}_T$. It is given by

$$\mathbf{r}_{EB} = \begin{vmatrix} \dfrac{\omega_0^2}{\omega_0^2 - \Omega_e^2} & \dfrac{-i\omega_0\Omega_e}{\omega_0^2 - \Omega_e^2} & 0 \\ \dfrac{i\omega_0\Omega_e}{\omega_0^2 - \Omega_e^2} & \dfrac{\omega_0^2}{\omega_0^2 - \Omega_e^2} & 0 \\ 0 & 0 & 1 \end{vmatrix} \frac{e\mathbf{E}_0}{m_e\omega_0^2}. \qquad (3)$$

In (3) E_0 is the complex electric field amplitude of the pump in vacuum, and e and m_e the charge and mass of electrons, respectively.

In the case of parametric *decay* processes [$e^{(+n)}$ is then neglected in (1)] the parametric dispersion relation (1) pre-

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dicts the excitation of a high-frequency mode with frequency ω_H and low-frequency mode with frequency $\omega = \omega_L - \mathbf{k}_0 \cdot \mathbf{V}_H$, where ω_L is the low plasma eigenfrequency and $\mathbf{V}_H = -(\partial \operatorname{Re} \epsilon_H / \partial \mathbf{k}_H)/(\partial \operatorname{Re} \epsilon_H / \partial \omega_H)$ is the group velocity of the high-frequency mode. In what follows, however, this finite-wavelength low-frequency shift will not play a significant role because we shall be interested in the excitation of high-frequency magnetized plasma modes with $\mathbf{V}_H \perp \mathbf{k}_0$. From (1) the parametric decay growth rate $\gamma \equiv \gamma_d$, for weakly damped waves, is found to be

$$\gamma_d^2 = [\langle e^{(-n)} \rangle - \langle e_{\rm thr}^{(-n)} \rangle][1 + \operatorname{Re} \chi_e(\omega, \mathbf{k})]^2 \times \left(\frac{\partial \operatorname{Re} \epsilon_L}{\partial \omega}\right)^{-1}_{\omega = \omega_L} \left(\frac{\partial \operatorname{Re} \epsilon_H}{\partial \omega}\right)^{-1}_{\omega = \omega_H}. \quad (4)$$

Here $e_{\rm thr}^{(-n)}$, given later in (10), is the threshold value of the coupling coefficient. Expression (4) is obtained under the assumption that $\gamma \gg \gamma_H, \gamma_L$, where γ_H and γ_L are linear damping rates of high- and low-frequency modes given by $\gamma_H = -(\operatorname{Im} \epsilon_H)/(\partial \operatorname{Re} \epsilon_H / \partial \omega_H)$, and $\gamma_L = -(\operatorname{Im} \epsilon_L)/(\partial \operatorname{Re} \epsilon_L / \partial \omega_L)$, respectively; in this case $e^{(-n)} \gg e_{\rm thr}^{(-n)}$. For $e_{\rm thr}^{(-n)} = 0$, the expression (4) will be referred to as the dissipation free parametric growth rate γ_{d0}. For the case $\gamma_H < \gamma < \gamma_L$ the parametric growth rate could be obtained by replacing γ_d^2 in (4) with $\gamma\gamma_L$. This is the case, for example, in isothermal plasmas ($T_e \sim T_i$) typical of tokamaks when the low-frequency plasma mode is heavily damped (quasi-mode) i.e., $\gamma_L \sim \omega_L$.

For the high-frequency dielectric permittivity appearing in (1), in the long-wavelength region ($k^2\rho_e^2 \ll 1$, $k^2 r_{\rm De}^2 \ll 1$, ρ_e the electron Larmor radius, $r_{\rm De}$ the electron Debye radius), we have the following relation[11]:

$$\epsilon_H(\omega, \mathbf{k}) = 1 - \frac{\omega_{pi}^2 \cos^2\theta}{\omega^2} - \frac{\omega_{pi}^2 \sin^2\theta}{\omega^2 - \Omega_i^2} - \frac{\omega_{pe}^2 \cos^2\theta}{\omega^2} - \frac{\omega_{pe}^2 \sin^2\theta}{\omega^2 - \Omega_e^2} + i\epsilon_{\rm coll}. \quad (5)$$

The collisional dissipation of magnetized plasma eigenmodes is given by

$$\epsilon_{\rm coll} = \frac{\omega_{pe}^2 \nu_{ei}}{\omega^3}\left(\cos^2\theta + \sin^2\theta \frac{\omega^2(\omega^2 + \Omega_e^2)}{(\omega^2 - \Omega_e^2)^2}\right). \quad (6)$$

If $\Omega_e \gtrsim \omega_{pe}$ the cold magnetized plasma modes are

$$\omega_H \approx \{\Omega_e[1 + (\omega_{pe}^2/2\Omega_e^2)\sin^2\theta];\omega_{pe}|\cos\theta|\} \equiv \omega_\pm. \quad (7)$$

Parametric excitation of eigenmode $\omega_- = \omega_{pe}|\cos\theta|$ [Gould–Trivelpiece, (G–T) mode or oblique Langmuir wave] and the corresponding heating of the plasma by this wave was already considered in Refs. 5, 6, and 12 and will not be treated here. It should be noted, however, that in the case of tokamak plasmas with $\omega_{pe} > \Omega_e$ the frequency curves $\omega_{\rm UH}(x)$, $\Omega_e(x)$, and $\omega_{pe}(x)$ [Fig. 1(a)] are overlapped and consequently the shaded area on the tokamak cross section [Fig. 1(b)] presents the region where simultaneous parametric excitation of the upper-hybrid wave (UH), the electron cyclotron wave (EC), and G–T modes can occur.

In the case of nearly perpendicular propagation, ω_+ is reduced to the upper-hybrid mode $\omega_{\rm UH}^2 = \omega_{pe}^2 + \Omega_e^2$. Perpendicular propagation in this frequency region also involves electron Bernstein modes with a frequency spectrum given, approximately, by[11]

$$\omega_{EB} \approx n|\Omega_e|\{1 + [I_n(k^2\rho_e^2)/k^2 r_{\rm De}^2]e^{-k^2\rho_e^2}\}, \quad (8)$$

and associated dielectric permittivity

$$\epsilon_{EB}(\omega, \mathbf{k}) = 1 - \frac{\omega_{pe}^2}{\omega}\sum_{n=-\infty}^{+\infty} \frac{n^2 A_n(k^2\rho_e^2)}{k^2\rho_e^2(\omega - n\Omega_e)} + in!2^n \frac{k^2 r_{\rm De}^2}{(k^2\rho_e^2)^n}\frac{\nu_{ei}}{n|\Omega_e|}, \quad (9)$$

where the last term, due to collisions, is evaluated for $k^2\rho_e^2 < 1$.

In the low-frequency domain there exist high-frequency ion-sound waves $IS(H)$ [$\omega_L = kV_s, V_s = (T_e/m_i)^{1/2}$], low-frequency ion-sound waves $IS(L)$ ($\omega_L = kV_s \cos\theta$), ion-Bernstein waves ($IB$) ($\omega_L = m|\Omega_i|\{1 + [I_m(k^2\rho_i^2)/k^2 r_{{\rm D}i}^2]e^{-k^2\rho_i^2}\}$), and lower-hybrid waves (LH) [$\omega_L \sim \omega_{pi}(1 + \omega_{pe}^2/\Omega_e^2)^{-1/2}$].

In what follows we shall be interested in the parametric excitation, by ordinary (O) and extraordinary (X) driver pumps, of modes with frequencies equal to the local electron

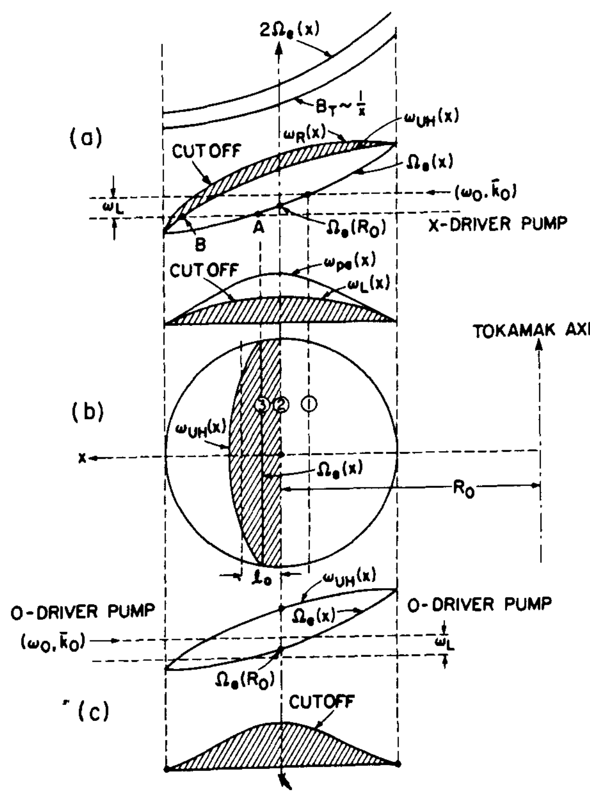

FIG. 1. Driver pump frequencies (X and O modes) in terms of x (distance from the tokamak axis) are shown. On the cross section of the [Fig. 1(b)] tokamak, a region of anomalous absorption of driver pump energy is shown. "A" denotes [Fig. 1(a)] a parametrically excited mode with local electron cyclotron frequency and "B" a parametrically excited upper-hybrid wave. Anomalous absorption length is denoted by l_a.

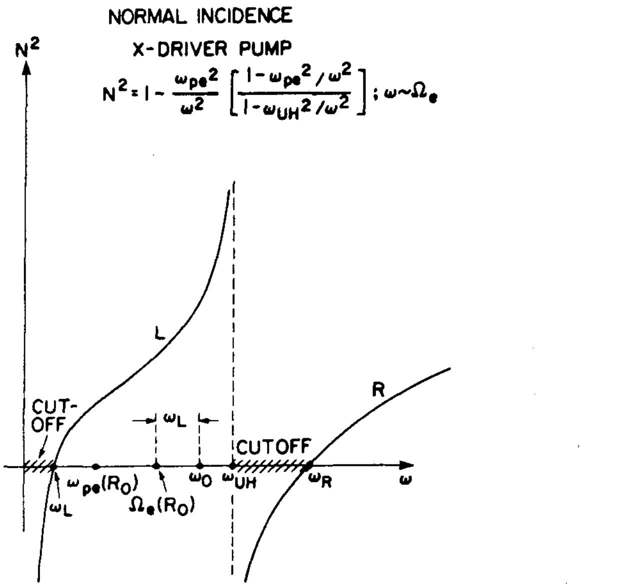

FIG. 2. Index of refraction $N = ck_0/\omega_0$ (k_0 is the wavenumber of the driver pump), for right-hand and left-hand polarized X-driver pump is shown for the case of strongly magnetized plasmas: $\Omega_e^2(x) \equiv (1+y)\omega_{pe}^2(x)$, where $0<y<1$. Here $\omega_L(x) = [\omega_{pe}(x)/2](\sqrt{5+y} - \sqrt{1+y})$ is the left-hand cutoff frequency, $\omega_R(x) = [\omega_{pe}(x)/2](\sqrt{5+y} - \sqrt{1+y})$ is the right-hand cutoff frequency, and $\omega_{UH}(x) = \omega_{pe}(x)\sqrt{2+y}$. Low-frequency parametrically excited waves are indicated by ω_L. R_0 is the major radius of the torus.

cyclotron frequency coupled to the above mentioned low-frequency modes. Accessibility of these modes in a tokamak is shown in Figs. 1, 2, and 3.

III. PARAMETRIC THRESHOLDS IN THE ECFR

From the parametric dispersion relation (1), the threshold value for decay processes is found to be

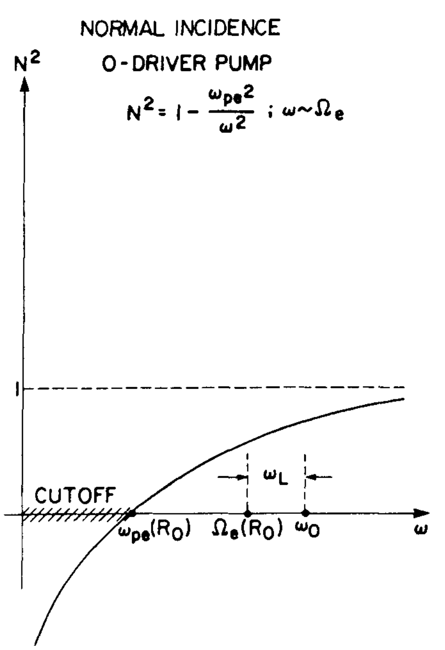

FIG. 3. Index of refraction for the O mode. Other quantities are the same as in Fig. 2.

$\{e_{\rm thr}^{(-n)}\}$

$$= \gamma_H \gamma_L \frac{1}{[1 + \operatorname{Re}\chi_e(\omega,\mathbf{k})]^2}\left(\frac{\partial\epsilon_H}{\partial\omega}\right)_{\omega=\omega_H}\left(\frac{\partial\epsilon_L}{\partial\omega}\right)_{\omega=\omega_L} \tag{10}$$

This expression gives us the so-called dissipative threshold of convective parametric instabilities in homogeneous magnetized plasmas. The dissipative threshold values for absolute parametric instabilities are higher, and given by [13,14]

$$\gamma_{d0} = [\gamma_H|V_L| + \gamma_L|V_H|]/2\sqrt{|V_H||V_L|} \equiv \gamma_a, \tag{11}$$

where γ_{d0} is γ_d of (4) with $e_{\rm thr} = 0$. Here V_H and V_L are group velocities of high- and low-frequency parametrically excited modes.

Taking into account linear dephasing due to weak inhomogeneity of the plasma, the conditions that the linear instabilities evolve to large amplitudes are [14,15a]

$$\gamma_{d0} \gg (2\gamma_a/\pi) \quad \text{and} \quad (\pi\lambda) \gg 1, \tag{12a}$$

for initially evolving absolute instabilities, and

$$\gamma_{d0} \gg 2(\gamma_H + \gamma_L)/\pi \quad \text{and} \quad (\pi\lambda) \gg 1, \tag{12b}$$

for initially evolving convective instabilities. Here $\lambda \equiv \gamma_{d0}^2/\kappa'|V_H V_L|$, where $\kappa' \equiv (d/dx)[k_0(x) - k_H(x) - k_L(x)]$, gives the spatial rate of dephasing.

In the important practical case of absolute parametric instabilities evolving in an inhomogeneous plasma and within a pump of finite extent, the threshold involves three conditions, giving both upper and lower bounds on the finite extent of the pump, and a lower bound on the dephasing due to the inhomogeneity. For a Gaussian-shaped pump having a maximum coupling γ_{d0} and effective extent $2L$ [i.e., $\gamma_{d0}\exp(-x^2/L^2)$], where $\pm x$ is along the direction of the (oppositely directed) group velocities, and including linear dephasing due to weak inhomogeneity of the plasma, the three conditions give approximately[14,15b]

$$\frac{\kappa' L}{\alpha_0} < 2.5; \quad \text{i.e., } \gamma_{d0} > \frac{\kappa' L\,|V_H V_L|^{1/2}}{2.5}, \tag{13a}$$

$$\alpha_0 L > 1; \quad \text{i.e., } \gamma_{d0} > \frac{|V_H V_L|^{1/2}}{L}, \tag{13b}$$

$$\frac{\kappa'}{\alpha_0^2} \lesssim 1.5; \quad \text{i.e., } \gamma_{d0} \gtrsim \left(\frac{\kappa'|V_H V_L|}{1.5}\right)^{1/2}, \tag{13c}$$

where $\alpha_0 \equiv \gamma_{d0}/|V_H V_L|^{1/2}$, and where each condition is also expressed to show explicitly the requirement on γ_{d0}, i.e., maximum pump amplitude. Note that the last condition (13c) is essentially the same as the second condition in (12).

In what follows we shall be interested in bulk plasma heating, which is realized through collisional dissipation of the high-frequency modes. If the dissipation of high-frequency modes is noncollisional (Landau or electron-cyclotron damping) the threshold value increases exponentially. For the decay wavenumbers k_d satisfying the following condition: $k_d \ll k_{\rm coll}$, where

$$k_{\rm coll}^2 r_{\rm De}^2 \approx \frac{\Omega_e^2}{\omega_0^2 - \omega_{pe}^2 - \Omega_e^2}\left[\ln\left(\frac{\omega_0}{\nu_{ei}}\right)^2\right]^{-1}, \tag{14}$$

TABLE I. Decay wavenumbers of primary excitations for parametric channels in the ECFR. The decay wavenumber is a function of layer position in real space [Fig. 1(b)], i.e., $k_d = k_d[\omega_0 - \Omega_e(x)]$. Consequently, different layers will have different decay wavenumbers. The cold and hot modes are defined by (34)–(37).

Primary decay processes	Decay wavenumber "cold" modes	"hot" modes
$\omega_0 = \omega_{UH} + \omega_{LH}$	$\left(\frac{k_\parallel}{k}\right)_d^2 = \frac{2(\omega_0 - \omega_{UH})}{\omega_{LH}}\left(\frac{m_e}{m_i}\right)$	$k_d^2\rho_e^2 = \frac{2}{3}\frac{\omega_{LH}}{\Omega_e\Omega_i}\frac{T_e}{T_i}(\omega_0 - \omega_{UH})$
$\omega_0 = \omega_{EB} + \omega_s;\omega_{LH};\omega_{IC}$	$k_d\rho_e = \left(\frac{2\Omega_e(\omega_0 - \Omega_e)}{\omega_{pe}^2}\right)^{1/2}$	$k_d\rho_e \ll 1,\ n = 1$
	$k_d\rho_e = \left(2^n n!\,\frac{\omega_0 - n\Omega_e}{\omega_0}\,\frac{\Omega_e^2}{\omega_{pe}^2}\right)^{1/2(n-1)}$	$n \neq 1$
$\omega_0 = \omega_{UH} + \omega_s$	$k_d = (\omega_0 - \omega_{UH})/V_s$	$V_s = (T_e/m_i)^{1/2}$

dissipation of high-frequency modes is dominantly collisional. In Table I we give decay wavenumbers for variety of parametric excitations.

From (10) and (2) ($k_0 \sim 0$ in the case of primary excitations) for principal harmonic excitations ($n = 1$) and for tokamak parameters $n_e \approx 10^{12}\ \text{cm}^{-3}$, $T_e \approx 200\ \text{eV} - 1\ \text{keV}$, $f_0 = (\omega_0/2\pi) \gtrsim [\Omega_e(R_0)]/(2\pi) = 10$–$40$ GHz one finds dissipative threshold electric fields in the range $E_0 \approx 30 - 120$ V/cm. The corresponding power flux is $I_{0,thr} = 2 - 50\ \text{W/cm}^2$ and, if the area of electron resonance is $S \approx 100\ \text{cm}^2$, the threshold power is $P_0 = 200 - 5\times 10^3$ W. The threshold values taking into account plasma inhomogeneity [$\pi\lambda = 1$, see Eq. (12)] can be obtained from (10) by substituting γ_H and γ_L with V_H/L_d and V_L/L_d, respectively, where $L_d \equiv [\pi/(\kappa')_{x=R_0}]^{1/2}$. Taking, typically, $L_d \approx 10\lambda_0$ we find the threshold field due to plasma inhomogeneity is approximately an order of magnitude larger, i.e., $E_{0,\text{thr}} \gtrsim 1(\text{kV/cm})$, and hence $I_{0,\text{thr}} \gtrsim 1(\text{kW/cm}^2)$ and $P_{0,\text{thr}} > 100$ kW; these are typical in current experiments on tokamaks with plasma parameters in the above given range (Versator II, FT-1, ISX-B, PLT, T-10, and JFT-2). Comparing (13a) and (13c) we note that for absolute instability the pump power flux would have to be about (L/L_d) times the pump power flux threshold due to plasma inhomogeneity. Given that $L \gg L_d$, and that the pump power flux is just sufficient to overcome the dephasing effect due to inhomogeneity, parametric excitations will evolve as convective instabilities.

In the case of high-density and high magnetic field tokamaks with parameters in the range $n_e \sim 10^{14}\ \text{cm}^{-3}$, $f_0 \sim 100$–200 GHz, and $T_e \gtrsim 1$ keV (like Alcator C and T-15) the threshold power flux is $I_{0,\text{thr}} \gg 10\ \text{kW/cm}^2$. In current experiments the maximum power flux available is around 10 kW/cm^2. Consequently significant parametric processes at ECR are not to be expected for these tokamaks until the power flux is raised up by an order of magnitude.

IV. SECONDARY DECAY PROCESSES

As the saturation mechanism of the parametrically induced turbulent plasma state, we consider secondary decays of high-frequency magnetized plasma modes $\omega_{H_1}(\mathbf{k}_{H_1})$ into other high-frequency $\omega_{H_2}(\mathbf{k}_{H_2})$ modes coupled with low-frequency modes $\omega_{L_2}(\mathbf{k}_{L_2})$, etc. This mechanism, as it will be shown later, yields a high absorption efficiency. In the lower-hybrid frequency range (LHFR) it was used in Ref. 12. In the case of secondary decay processes the finite wavelength of the driver pump (high-frequency longitudinal mode) must be taken into account in contrast to the case of primary decays where the dipole approximation ($k_0 = 0$) is valid. The parametric mode–mode coupling dispersion relation (1) describing secondary decay processes has the following form (principal harmonic excitation $n = 1$):

$$\epsilon(\omega_{n+1}^L, \mathbf{k}_{n+1}^L)\epsilon(\omega_{n+1}^L - \omega_n^H, \mathbf{k}_{n+1}^L - \mathbf{k}_n^H) + \frac{k_{n+1}^2}{4}\frac{[(\mathbf{k}_{n+1}^L - \mathbf{k}_n^H)\mathbf{r}_{EB}^{(n)}]^2}{(\mathbf{k}_{n+1}^L - \mathbf{k}_n^H)^2}\chi_i(\omega_{n+1}^L, \mathbf{k}_{n+1}^L) \times [1 + \chi_e(\omega_{n+1}^L, \mathbf{k}_{n+1}^L)] = 0. \tag{15}$$

Here $(\omega_n, \mathbf{k}_n)$ are the frequency and the wave vector of the nth cascade, and $r_{EB}^{(n)}$ is the oscillation amplitude of electrons in the high-frequency electric field of the nth cascade [see (3)].

A. Absolute secondary decay instabilities

From (15), the parametric growth rate of the nth cascade is found to be

$$\gamma_n^2 = \frac{k_{n+1}^2}{4}\frac{[(\mathbf{k}_{n+1} - \mathbf{k}_n)\mathbf{r}_{EB}]^2}{(\mathbf{k}_{n+1} - \mathbf{k}_n)^2} \times [1 + \text{Re}\,\chi_e(\omega_{n+1}, \mathbf{k}_{n+1})]^2 \times \left(\frac{\partial\,\text{Re}\,\epsilon_L}{\partial\omega}\right)^{-1}_{\omega=\omega_L}\left(\frac{\partial\,\text{Re}\,\epsilon_H}{\partial\omega}\right)^{-1}_{\omega=\omega_H}. \tag{16}$$

Here $\omega_H \equiv \omega_H(\mathbf{k}_n^H - \mathbf{k}_{n+1}^L)$ and $\omega_L \equiv \omega_L(\mathbf{k}_{n+1}^L)$. Expression (16) is obtained under the assumption that $\gamma_n > \gamma_n^H, \gamma_n^L$, where γ_n is the temporal growth rate of the secondary decay instabilities. In the model of a homogeneous plasma, if $V_H V_L < 0$ the parametric decay instabilities are absolute if[13,14] [see (11)]

$$\tfrac{1}{2}\gamma_L(V^H/V^L)^{1/2} < \gamma_n < \omega_L. \tag{17}$$

In (17) the demand for the discrete (line-like) turbulence spectrum ($\gamma_n < \omega_L$) is also taken into account. Due to the inhomogeneity of tokamak plasmas and for weaker driver pumps, parametrically excited high-frequency instabilities could become convective and, consequently, the instability description by a temporal growth rate is not adequate.

B. Convective secondary decay instabilities

If $V_H V_L > 0$ the energy of excited waves is effectively convected away from the region of primary excitation. Consequently, processes in that region may be linear with respect to electric field amplitudes of excited waves. However, on the length $l \sim 1/(\mathrm{Im}\, k_H)$ amplification of the high-frequency mode electric field could be significant and nonlinear dissipation of the modes dominant. The maximum spatial growth rate is

$$\mathrm{Im}\, k_H = \gamma_{d0}/|V_L V_H|^{1/2}. \tag{18}$$

Here γ_d is the dissipation-free temporal growth rate and V_H and V_L group velocities evaluated at $\mathbf{k} = \mathbf{k}_0 - \mathbf{k}_d$ and $\mathbf{k} = \mathbf{k}_d$, respectively. (In Sec. II, $\mathrm{Im}\, k_H$ was identified as α_0.)

The expression (18) is valid if the excited modes are weakly damped, ie., $\gamma > \gamma_L, \gamma_H$. In the case of excitation of low-frequency quasimodes (LFQM) the inequality $\gamma_H < \gamma < \gamma_L$ is satisfied. Then, for strong pumps,[21]

$$\mathrm{Im}\, k_H = (\gamma \gamma_L / V_H V_L)^{1/2}. \tag{19}$$

Note that in the case of very high group velocities of high-frequency modes amplification occurs on a large distance which could be larger than the dimensions of interaction region[14] or even than the (radial) dimensions of the tokamak plasma. In that case nonlinear dissipation of excited waves does not play any significant role. However, we shall be interested in relatively slow waves, longitudinal waves, and their saturation distance l_a (l_a is the anomalous absorption length to be defined in Sec. V C).

V. NONLINEARLY ABSORBED ENERGY

In the model of cascade saturation the nonlinearly absorbed power density is given by

$$Q^{\mathrm{abs}} = 2\gamma_0(E_0) W_1, \tag{20}$$

in the case of absolute instabilities, or by

$$Q^{\mathrm{conv}} = 2\, \mathrm{Im}[k_0(E_0)]\, V_H W_1, \tag{21}$$

in the case of convective instabilities. Here $\gamma_0(E_0)$ and $\mathrm{Im}[k_0(E_0)]$ are primary temporal and spatial growth rates, respectively, and $W_1(\omega_1, k_1^H)$ represents the energy contained in the first cascade of the high-frequency mode and is given by [13]

$$W_1(\omega_1, k_1^H) = \left(\frac{\partial}{\partial \omega}[\omega\, \mathrm{Re}\, \epsilon^H(\omega, \mathbf{k})]\right)_{\omega = \omega_1(\mathbf{k}_1^H)} \frac{|\mathbf{E}(\mathbf{k}_1^H, \omega_1)|^2}{8\pi}. \tag{22}$$

In order to find Q, as is evident from (20) and (21), we must know the turbulence spectrum $W_n(\mathbf{k}, \omega)$ and particulary $W_1(\mathbf{k}_1^H, \omega_1)$. The answer can be obtained from the turbulence theory based on cascading saturation mechanism. A complete development of turbulence theory based on nonlinear kinetic equations in the LHFR is given in Ref. 12. Here, however, we shall give a simple power-balance formalism of turbulence processes in the ECFR based on the cascade saturation mechanisms.

A. Power balance theory of cascading processes in the ECFR

The cascade saturation mechanism was first used in Ref. 16 for isothermal plasmas and later in Ref. 17 for consideration of nonisothermal plasmas. A power balance theory for isotropic plasmas was proposed in Ref. 18, and for anisotropic plasmas in Ref. 6.

We shall consider a quasistationary turbulence spectrum consisting of N cascades with frequency width $\delta\omega \sim \max(\gamma_n, \gamma_H, \gamma_L)$. In the weak turbulence theory $\gamma_n \ll \omega_L$ (and $\gamma_L \lesssim \omega_L$) the turbulence spectrum can be considered as discrete, consisting of N discrete lines located at frequency intervals equal to ω_L. In the case $\gamma \lesssim \omega_L$, a ponderomotive force affects the low-frequency mode dispersion features and the mode–mode coupling dispersion relation (1) is no longer valid. Consequently a strong coupling of excited waves occurs. For $\gamma \gtrsim \omega_L$ the turbulence spectrum is continuous and power-balance theory is not applicable. For high temperature tokamak plasmas $T_e > 1$ keV the inequality $\gamma_H \ll \gamma_L$ is always valid and consequently the turbulent level of low-frequency modes is significantly less in comparison with the turbulent level of the high-frequency modes. Accordingly, it could be considered that the low-frequency wave energy is completely dissipated through the linear Landau damping mechanism (or linear cyclotron damping). This becomes more pronounced for isothermal plasmas $[\gamma_s \sim \omega_s \sim k V_{Ti}, V_{Ti} = (T_i/m_i)^{1/2}]$ when the dispersion relation (1) describes processes of induced scattering of a driver pump (external electric field or longitudinal high-frequency plasma mode) by ions or electrons.

The stationary turbulence spectrum of the high-frequency magnetized plasma modes, in the case of absolute instabilities, is described by

$$\gamma_{n-1}(W_n^H + W_n^L) = \gamma_H W_n^H + \gamma_L W_n^L + \gamma_n (W_{n+1}^H + W_{n+1}^L), \quad n \geqslant 1. \tag{23}$$

Here $\gamma_{n-1}(W_n^H + W_n^L)$ represents the power contained in the nth cascade which is partly, linearly dissipated collisionally in the high-frequency mode $\gamma_H W_n^H$ and noncollisionally in the low-frequency mode $\gamma_L W_n^L$. The flow of energy into $(n+1)$th high- and low-frequency cascade is accounted for by $\gamma_n (W_{n+1}^H + W_{n+1}^L)$.

In the case of convective instabilities the power balance equation takes the following form:

$$\mathrm{Im}\, k_{n-1} V_H (W_n^H + W_n^L) = \gamma_H W_n^H + \gamma_L W_n^L + \mathrm{Im}\, k_n V_H (W_{n+1}^H + W_{n+1}^L), \quad n \geqslant 1. \tag{24}$$

The connection between the energy content in low- and high-frequency parametrically excited modes is given by

$$W_n^L = (N - n + 1)\frac{\gamma_H}{\gamma_L}\frac{\omega_L}{\omega_H} W_n^H, \quad n = 1 \ldots N. \tag{25}$$

It is evident from (25) that W_n^L could be neglected in (23) and (24). In (25), N is the total number of cascades. Substituting the expression for the parametric temporal growth rate (4) into (23), or the spatial growth rates (18), (19), into (24), we can obtain a turbulence spectrum of high-frequency modes. In the case of convective instabilities we have

$$E_n = \frac{\gamma_H}{\mathrm{Im}\, k_0 V_H} E_0 \left(\frac{\mathrm{Im}\, k_0 V_H}{\gamma_H} - \frac{n}{3} \right), \quad n = 0 \ldots N. \tag{26}$$

In (26) $\gamma_n > \gamma_H, \gamma_L$ is assumed, E_n is the electric field amplitude of nth cascade, and E_0 is the free-space electric field amplitude of the driver pump. From (26) with $(E_N = E_{\mathrm{thr}}, E_0 \gg E_{\mathrm{thr}})$ we obtain, for the total number of cascades,

$$N \sim 3 \frac{\mathrm{Im}\, k_0 |V_H|}{\gamma_H}. \tag{27}$$

Putting $n = 1$ in (26) we obtain

$$E_1 \lesssim E_0. \tag{28}$$

This result is, however, only valid in the case when the primary growth rate $\gamma_0(E_0)$ is of the same nature as the secondary one $\gamma_1(E_1)$. [This was assumed in obtaining (26).] The more general case is when the primary parametric growth rate is not of the same nature as the secondary one; for example when an e.m. wave decays into high- and low-frequency longitudinal modes and the high-frequency mode decays, further, into two high-frequency modes; this is possible in the short-wavelength region $(k\rho_e \gg 1)$ of second harmonic electron Bernstein mode $(2\Omega_e \to \Omega_e + \Omega_e)$. In this paper such secondary decays will not be treated.

Equation (23) (for $n = 1$) could be solved by means of the approximation $W_1 \approx W_2$ [the error is minimum $\Delta = W_1 - W_2$, $W_1 = W_n + (n-1)\Delta$]. Then $\gamma_0(E_0) \approx \gamma_1(E_1)$ is obtained. From this equation we can find

$$E_1 = f(E_0). \tag{29}$$

Note that (29) is obtained under the assumption of negligible linear dissipation of the high-frequency cascades $(\gamma_0 \gg \gamma_H)$ for the case of absolute instabilities [Eq. (23)], and in the case of convective instabilities [Eq. (24)] under the assumption that $(1/\mathrm{Im}\, k_1^H) \ll V_H/\gamma_H$, i.e., that the amplification length of the primary excited high-frequency mode is less than its linear damping length. This is realized if the driver pump electric field is strong enough.

For the absorbed energy, from (29) and (20), we obtain

$$Q^{\mathrm{abs}} = 2\gamma_1(E_1)\{ [f(E_0)]^2/4\pi \}. \tag{30}$$

From (30) we see how the secondary decay processes play a determining role in the energy absorption. In what follows we shall be interested in the cases when primary and secondary decays are of the same nature, so that

$$Q^{\mathrm{abs}} = 2\gamma_1(E_1)(E_0^2/4\pi). \tag{31}$$

If the low-frequency modes are weakly damped $(\gamma > \gamma_L)$, we obtain from (20), (21), and (18)

$$Q^{\mathrm{conv}} = (V_H/V_L)^{1/2} Q^{\mathrm{abs}}, \quad \gamma > \gamma_H, \gamma_L. \tag{32}$$

In the case of heavily damped low-frequency modes (quasi-modes) the corresponding relation could be obtained from (20), (21), and (19)

$$Q^{\mathrm{conv}} = (\gamma_L V_H/\gamma V_L)^{1/2} Q^{\mathrm{abs}}, \quad \gamma_H < \gamma < \gamma_L. \tag{33}$$

The expressions (30)–(33) give the relations between the energies nonlienarly absorbed (through cascading processes) by convective and absolute instabilities.

In the case of laser–plasma interactions, convective instabilities (due to the inhomogeneity of plasma) always enhance [see (32)] the absorption with respect to the absolute instability. In the ECFR in the case of "cold" modes:

$$\omega^{UH} = \omega_{uh} \left(1 - \frac{1}{2} \frac{\omega_{pe}^2 \Omega_e^2}{\omega_{uh}^4} \frac{k_\parallel^2}{k^2} \right), \tag{34}$$

where $\omega_{uh}^2 = \omega_{pe}^2 + \Omega_e^2$, and

$$\omega^{LH} = \omega_{lh} \left(1 + \frac{m_i}{m_e} \frac{k_\parallel^2}{k^2} \right)^{1/2}, \tag{35}$$

where $\omega_{lh} = [\omega_{pi}/(1 + \omega_{pe}^2/\Omega_e^2)^{1/2}]$. For these modes $V^{UH}/V^{LH} \approx (m_e/m_i)^{1/2}(\omega_{pe}/\Omega_e) \ll 1$ is obtained. If "hot" modes are excited

$$\omega^{UH} = \omega_{uh} \left(1 - \frac{1}{2} \frac{\omega_{pe}^2}{\omega_{uh}^2} k_1^2 \rho_e^2 \right), \tag{36}$$

$$\omega^{LH} = \omega_{lh} \left(1 + \frac{3}{2} k_1^2 \rho_e^2 \frac{\Omega_e \Omega_i}{\omega_{pi}^2} \frac{T_i}{T_e} \right), \tag{37}$$

and the group velocities ratio is $(V^{UH}/V^{LH}) \approx (V^{EB}/V^{LH}) \approx \frac{1}{3}(\omega_{pe}/\Omega_e)^2(\omega_{pi}/\Omega_i)(T_e/T_i) > 1$, $(\omega_{pi} \gg \Omega_i)$. Hence in the case of excitation of "hot" modes (convectively unstable) the nonlinear absorption of energy is also enhanced.

B. Anomalous absorption frequency

In the cascade saturation mechanism energy transferred from the external pump into the first cascade is collisionally dissipated through N further cascades. The condition for collisional dissipation of the high-frequency mode (in a high-temperature plasma it is a weak linear dissipation) is essential because only in this case is a significant turbulence level of high-frequency modes achieved. In such a situation the nonlinear dissipation of parametrically excited high-frequency modes is dominant. The opposite case is encountered with low-frequency modes where a strong linear dissipation is assumed (linear Landau damping or cyclotron damping); in this case the appearance of low-frequency parametric turbulence is excluded.

Based on the law of conservation of energy we can write

$$\nu_a W_0 = 2\gamma_0(E_0) W_1 = 2 \sum_{n=1}^{n=N} \gamma_H W_n, \quad \gamma_H \sim \nu_{ei}. \tag{38}$$

The expression (38) is to be considered as a definition of anomalous absorption (collision) frequency. As $W_n \sim W_{n+1}$, $n > 0$ (as a rule in this saturation mechanism), then $\gamma_1(E_1) \sim \gamma_0(E_0)$ which, if the primary and secondary decays are of the same nature, gives (with $W_1 \lesssim W_0$)

$$\nu_a \sim 2\gamma_1(E_1) \sim 2\gamma_0(E_0). \tag{39}$$

From (39) it is evident that the anomalous absorption frequency exceeds the electron–ion collision frequency by a factor of $2[\gamma_0(E_0)]/\nu_{ei}$. In the case of the excitation of low-frequency quasimodes ($T_e \sim T_i$, typical for tokamak plasmas) the anomalous absorption frequency has a quadratic depen-

TABLE II. Anomalous absorption lengths for a variety of decay channels. The geometry employed in evaluating the quantities is as follows: $\mathbf{B}_0 = (0, 0, B_t)$, $\mathbf{k}_0 = (k_0, 0, 0)$, $\mathbf{E}_0 = \{0, E_0, 0\}$ for X mode, and $\mathbf{E}_0 = \{0, 0, E_0\}$ for O mode. The wave vector of excited waves is in the (y, z) plane. Here κ is Boltzmann's constant. The values for l_a are obtained utilizing expressions (40), (31), and (16).

Driver pump frequency	Decay waves	O mode	(Normal incidence) X mode
$\omega_0 = \omega_{EB} + \omega_{IS(H)}$ $\omega_{EB} = n\lvert\Omega_e\rvert\left(1 + \frac{I_n(k^2\rho_e^2)}{k^2 r_{De}^2} e^{-k^2\rho_e^2}\right)$ $\Omega_e \gtrsim \omega_{pe}$ $\omega_{IS(H)} = kV_s$	$EB + IS(H)$	Decay is not possible	$l_a = \frac{c}{2}\frac{4\Omega_e(4\pi n_e\kappa Te)^{1/2}}{(\Omega_e\omega_s)^{1/2}\omega_{pe}E_0}, \quad n=1$ $l_a = \frac{c}{2}\frac{4\Omega_e^3}{\omega_{pe}^3}\frac{2(n-1)(4\pi n_e\kappa T_e)^{1/2}}{(\Omega_e\omega_s)^{1/2}E_0}, \quad n>1$
$\omega_0 = \omega_{EB} + \omega_{IB}$ $\omega_{IB} = m\lvert\Omega_i\rvert\left(1 + \frac{I_n(k^2\rho_i^2)}{k^2 r_{Di}^2} e^{-k^2\rho_i^2}\right)$	$EB + IB$	Decay is not possible	$l_a = \frac{c}{2}\frac{2(k_d^2 r_{De}^2)^2 k_d^2 r_{Di}^2\omega_{pi}\omega_{pe}2^{(m+n)/2}(m!n!)^{1/2}}{\mu_n mn(\Omega_i\Omega_e)^{3/2}(k_d^2\rho_i^2)^{(m+1)/2}(k_d^2\rho_e^2)^{(n+1)/2}}$ $\mu_1^2 = \frac{k^2e^2E_0^2}{m_e^2\Omega_e^4}\left(\frac{\Omega_e}{\omega_{pe}}\right)^4, \quad n=1; \quad \mu_n^2 = \mu_1^2\frac{\omega_{pe}^4}{\Omega e^4}\frac{1}{4(n-1)^2}, \quad n>1$
$\omega_0 = \omega_{EB} + \omega_{LH}$ $\omega_{LH} = \omega_{pi}\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{-1/2}$	$EB + LH$	Decay is not possible	$l_a = \frac{c}{2}\frac{4\Omega_e}{(\omega_{LH}\Omega_e)^{1/2}\omega_{pe}k_d r_{De}}\frac{(4\pi n_e\kappa Te)^{1/2}}{E_0}, \quad n=1$ $l_a = \frac{c}{2}\frac{[8\cdot 2^n(n-1)!]^{1/2}\Omega_e k_d r_{De}2(n-1)}{(\omega_{LH}\Omega_e)^{1/2}\omega_{pe}(k_d^2\rho_e^2)(n+1)/2}\frac{(4\pi n_e\kappa Te)^{1/2}}{E_0}, \quad n>1$
$\omega_0 = \omega_{OEC} + \omega_{IS(H)}$ $\omega_{OEC} = \Omega_e\left(1 + \frac{1}{2}\frac{\omega_{pe}^2}{\Omega_e^2}\sin^2\theta\right)$	$OEC + IS(H)$	$l_a = \frac{c}{2}\frac{8\Omega_e^3}{\omega_{pe}^3}\frac{(4\pi n_e\kappa Te)^{1/2}}{\sin 2\theta(\Omega_e k_d V_s)^{1/2}E_0}$	$l_a = \frac{c}{2}\frac{4\Omega_e(4\pi n_e\kappa Te)^{1/2}}{(\Omega_e\omega_s)^{1/2}\omega_{pe}E_0}$
$\omega_0 = \omega_{OEC} + \omega_{IS(L)}$ $\omega_{IS(L)} = kV_s\cos\theta$	$OEC + IS(L)$	$l_a = \frac{c}{2}\frac{(4\pi n_e\kappa Te)^{1/2}}{(\Omega_e k_d V_s\cos\theta)^{1/2}(\cos\theta\sin 2\theta)E_0}$	$l_a = \frac{c}{2}\frac{4\Omega_e(4\pi n_e\kappa Te)^{1/2}}{\omega_{pe}(\Omega_e k_d V_s)^{1/2}(\cos^{3/2}\theta)E_0}$
$\omega_0 = \omega_{UH} + \omega_{IS(H)}$ $\omega_{UH} \sim (\omega_{pe}^2 + \Omega_e^2)^{1/2}$	$UH + IS(H)$	Decay is not possible	$l_a = \frac{c}{2}\frac{4k_d r_{De}}{\mu_x}\frac{\omega_{UH}}{\omega_{pe}(k_d V_s\omega_{UH})^{1/2}}$ $\mu_x = k_d r_{De}\frac{E_0}{(4\pi n_e\kappa Te)^{1/2}}$
$\omega_0 = \omega_{UH} + \omega_{LH}$ $\omega_{LH} = \omega_{pi}\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{-1/2}$	$UH + LH$	Decay is not possible	$l_a = \frac{c}{2}\frac{4}{\mu_x}\frac{\omega_{UH}}{\omega_{pe}(\omega_{UH}\omega_{LH})^{1/2}}$

dence on the external electric field $\nu_a \sim E_0^2$ [see (1) and (2) for $n = 1$].

C. Anomalous absorption length

The anomalous absorption frequency is to be taken as a local characteristic of nonlinear absorption processes. A high anomalous absorption frequency, however, does not mean, in general, a high nonlinear absorption efficiency. For this reason we shall define another characteristic quantity, namely, the anomalous absorption length as a distance (l_a) which is traversed by the external driver pump with group velocity V_{gr} in the time of nonlinear absorption $t_{NL} \sim (1/\nu_a)$. Accordingly,

$$l_a = V_{gr} [E_0^2 / 4\pi Q(E_1^2)]. \tag{40}$$

In the ECFR $V_{gr} \sim c$ (c is the speed of light in vacuum). Here, $Q(E_1^2)$ is either (31) or (32) for, respectively, absolute or convective instabilities.

In Table II we give the values of l_a for a variety of parametric excitations in the ECFR for the case $\gamma > \gamma_H, \gamma_L$. The corresponding expressions of anomalous absorption lengths for low-frequency quasimode excitation (ion-acoustic and lower-hybrid quasimode) could be obtained by substitution for l_a:$(cl_a/4k_d v_{Ti})^{1/2}$, for the ion-acoustic quasimode, and $(cl_a/4\omega_{LH})^{1/2}$ in the case of the lower-hybrid quasimode excitation.

It must be noted, however, that the influence of real conditions (plasma inhomogeneity, finite extent of driver pump, etc.) are indirectly included in (40) through $Q(E_1^2)$, i.e., through the secondary parametric growth rate. This is only approximate. A more accurate description would have to use an appropriate turbulence theory for an inhomogeneous plasma, including the finite extent of the driver pump.

From Table II, in the case of parametric excitation of electron-Bernstein modes coupled with lower-hybrid quasimodes, the anomalous absorption length is found to be $(n = 1)$

$$l_a = \frac{2}{\pi}\lambda_0 \frac{\beta}{k^2 r_{De}^2}\left(\frac{B_0}{E_0}\right)^2\left(\frac{\Omega_e}{\omega_{pe}}\right)^2, \quad k_d r_{De} \lesssim 1. \tag{41}$$

Here, λ_0 denotes the free-space wavelength of the driver pump and $\beta = (8\pi n_e T_e / B_0^2)$ is the "plasma beta." For reactor-type tokamaks with parameters $B_0 \approx 5$–10 T, $n_e \approx 10^{14}$ cm^{-3}, $T_e \approx 1$–5 keV, the anomalous absorption length is of the order of 10 λ_0 if the driver pump electric field is enormously strong $E_0 \lesssim 100$ kV/cm ($P_0 \gg 1$ MW). For plasma parameters $n_e \approx 2\times 10^{12}$ cm^{-3}, $T_e \approx 200$ eV, and $B_0 \approx 1$T (Versator II and FT-1) or $n_e \approx 2\times 10^{13}$ cm^{-3}, $T_e \approx 1$ keV, and $B_0 \approx 3$T (PLT) such an anomalous absorption length could be obtained if $E_0 \lesssim 10$ kV/cm. This estimate is based on the model of direct parametric excitation of the EB mode in region 3 [see Fig. 1(b)]. However, in linear theory the X mode can then propagate to the upper-hybrid resonance layer[19] where linear conversion takes place. Typical mode-conversion enhancement of the electric field is five times or more[20] so that parametric processes can be induced if $E_0 \approx 1$ kV/cm, which corresponds to a power flux $I_0 \approx 1$ kW/cm^2 readily available in experiments.[8]

It is to be noted that (41) is obtained for perpendicular propagation of an electron-Bernstein mode with pure collisional damping. In the presence of the tokamak poloidal magnetic field, a more realistic model of electron-Bernstein mode propagation must account for a finite k_z. In this case the linear dissipation of the electron-Bernstein mode is due to the Landau and cyclotron damping. We then have $\gamma_{EB} < \gamma_{LH} \sim \omega_{LH}$, and for the anomalous absorption length we find

$$l_a = \frac{c}{2}\left[\frac{1}{8}\frac{k_y^2}{k_y^2 + k_z^2}\left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \frac{1}{\beta}\left(\frac{E_0}{B_0}\right)^2 k^2 r_{De}\Omega_e - \gamma_{EB}\right]^{-1}. \tag{42}$$

In the regime far from the threshold $(E_0 \gg E_{0,\mathrm{thr}})\gamma_{EB}$ in (42) could be neglected and (42) is $(k_y^2 + k_z^2)/k_y^2$ times larger than (41). In the near-threshold regime, the influence of the noncollisional damping γ_{EB} is significant and l_a given by (42) is several times larger than (41). This means that the efficiency of nonlinear absorption for this case is reduced.

Let us now consider the launching of the ordinary mode from the low magnetic field side. As seen from Fig. 1(c), it reaches the upper-hybrid layer where the excitation of electron cyclotron waves, with wave vector $\mathbf{k}$ at an angle θ with respect to $\mathbf{B}_0$ [see (7)], coupled to low-frequency modes can occur. In the case of excitation of the lower-hybrid waves, we obtain for the anomalous absorption length

$$l_a = \frac{8}{\pi}\lambda_0 \frac{\beta}{k^2 r_{De}^2}\left(\frac{B_0}{E_0}\right)^2\left(\frac{\Omega_e}{\omega_{pe}}\right)^2 \frac{\gamma_{LH}}{\omega_{LH}} \frac{1}{(\sin 2\theta)^2}\left(\frac{\omega_0}{\omega_{pe}}\right)^2. \tag{43}$$

The minimum value for l_a is obtained for waves propagating at an angle $\theta \approx 45°$. From (43) it is also evident that the excitation of lower-hybrid quasimodes $(\gamma_{LH} \sim \omega_{LH})$ is not an effective channel for nonlinear dissipation of the external power. In the case of O mode launching from the low field side [see Fig. 1(c)] parametric processes can also occur in plasma with $n_e \approx 10^{12}$ cm^{-3}, $T_e \approx 1$ keV, and $B_0 \approx 1$ T if $I_0 \approx 1$ kW/cm^2. For these parameters, in the case of excitation of well-defined lower-hybrid waves $(\gamma_{\mathrm{LH}} \lesssim 10^{-2}\omega_{\mathrm{LH}})$, we obtain from (43) l_a values of $10\lambda_0$.

VI. COLLISIONAL AND NONCOLLISIONAL NONLINEAR DISSIPATION OF PARAMETRICALLY EXCITED MODES

The cascade saturation mechanism is based on linear collisional dissipation of high-frequency plasma modes. Consequently, the decay wavenumber k_d (see Table I) is to be located on the dispersion curve of corresponding parametrically excited modes where collisional dissipation is dominant. On the other hand the decay wavenumber is a function of "x" (the distance from the tokamak axis), i.e., $k_d = f[\omega_0 - \Omega_e(x)]$ [see Fig. 1(a)], so that as a function of "x," modes with different k_d will be excited. Through cascading processes this mode can enter the region of noncollisional linear dissipation, at which point its saturation cannot be treated by the cascading mechanism. In the case of an electron Bernstein mode with perpendicular propagation linear dissipation is purely collisional, while in the case of quasiperpendicular propagation there exist regions on its dispersion curves where noncollisional (Landau and cyclotron) dissipation is dominant (Ref. 11). For other modes in the ECFR such a noncollisional region always exists.

In order that collisional nonlinear dissipation be dominant, $N\omega_L$ is to be less than the hot-plasma dispersion correction of the corresponding modes. In the opposite case, a significant amount of energy is transferred into the noncollisional linear dissipation region and can lead to acceleration of electrons (tail plasma heating). In the upper limit of applicability of weak turbulence theory ($\gamma \sim \omega_L$) for bulk heating through cascading processes to be dominant γ^2/γ_H must be less than the hot-plasma dispersion correction of the corresponding modes. For the excitation of upper-hybrid waves, this condition gives

$$\gamma^2 \lesssim \tfrac{1}{8}\gamma_{UH}(\omega_{pe}^2/\omega_{UH})k_\perp^2\rho_e^2, \quad k_\perp^2\rho_e^2 < 1. \tag{44}$$

Note that in the case of very small k_d when $\omega_0 \sim \Omega_e(x)$ [which is realized in the vicinity of the line 1, Fig. 1(b)] the low-frequency mode has a zero frequency and aperiodic instabilities can take place. In this case other saturation mechanisms need to be studied. Decay instabilities and the cascading saturation mechanism take place in the shaded region [Fig. 1(b)] if inequality (44) is satisfied.

VII. CONCLUSION

In this paper we have been interested in nonlinear theory of parametric processes induced by the external driver pump in the electron cyclotron frequency range in tokamak plasmas. It was shown [see (41)–(43) and Table II] that in both cases of X- and O-mode launching parametric processes are unlikely in tokamaks like Alcator C and T-15 unless gyrotrons with power fluxes around 100 kW/cm^2 are used. The situation is different in tokamaks with parameters similar to those of Versator II, FT-1, and PLT where parametric processes and effective cascade saturation mechanisms take place if $I_0 \gtrsim 1$ kW/cm^2. A verification of the cascading mechanism presented would be the observation in experiments and/or simulations of discrete or semidiscrete turbulence spectra around Stokes line.

Finally, we remark that in linear absorption the pump energy is transferred to the electron component of the plasma and, if the confinement time is long enough, part of this energy is transferred to the ions. However, in nonlinear (parametric) absorption some of the pump energy can be directly transferred to the ions through the excited low-frequency modes. Further, cascade saturation processes lead to bulk plasma heating in contrast to tail heating which occurs with linear absorption. Typical linear absorption lengths are several centimeters and, as shown above, anomalous absorption lengths are of the same order if $I_0 \gtrsim 1$ kW/cm^2. The recent observations[8] of simultaneous parametric excitations and bulk plasma heating are consistent with our model of nonlinear absorption.

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