Reprint Series
27 January 1989, Volume 243, pp. 494–500

SCIENCE

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Recently, a strong research effort has been focused on applications of beat waves in plasma interactions. This research has important implications for various aspects of plasma physics and plasma technology. This article reviews the present status of the field and comments on plasma probing, heating of magnetically confined and laser plasmas, ionospheric plasma modification, beat-wave particle acceleration, beat-wave current drive in toroidal devices, beat wave–driven free-electron lasers, and phase conjugation with beat waves.

INTEREST IN NONLINEAR MIXING OF ELECTROMAGNETIC waves in plasmas (particularly beat-wave plasma interaction) began more than two decades ago with the theoretical paper by Kroll, Ron, and Rostoker (*1*). A pioneering experiment designed to test the theory, performed by Stansfield, Nodwell, and Meyer (*2*), marked the beginning of experimental studies in nonlinear optics in plasmas. The early motivations for the research were applications to the diagnosis and heating of laboratory plasmas (*3–8*) and to ionospheric modification (*9, 10*). Recently, much attention has been devoted to beat waves because of their application to particle acceleration (*11–13*), current drive in tokamaks (*14, 15*), free-electron lasers (FELs) (*16*), defense sciences (*17*), and phase conjugation (*18*). In addition, from the viewpoint of the basic plasma physics, it is interesting that the presence of a beat-wave driver leads

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to the appearance of new plasma instabilities, nonexistent in the case of a single-driver pump (*19*).

We start with some basic aspects of the theory of nonlinear mixing of electromagnetic waves in plasmas. We assume that the externally applied electric field is given in the form $\mathbf{E}(t) = \mathrm{Re}\mathbf{E}_a(t)$ (t is time and Re denotes the real part) with $\mathbf{E}_a(t)$, the analytical signal of $\mathbf{E}(t)$, given by

$$\mathbf{E}_a(t) = \mathbf{E}_0 e^{i\phi_0} e^{-i(\omega_0 t - \mathbf{k}_0 \cdot \mathbf{r})} + \mathbf{E}_1 e^{i\phi_1} e^{-i(\omega_1 t - \mathbf{k}_1 \cdot \mathbf{r})} \quad (1)$$

In Eq. 1 $\mathbf{E}_0,\mathbf{E}_1$ are, respectively, the amplitudes of the electric field in the principal and escort driver pump and ω_0,ω_1, $\mathbf{k}_0,\mathbf{k}_1$, and ϕ_0,ϕ_1 are the corresponding angular frequencies, wave vectors, and initial phases, respectively. The double-driver pump indices nonlinear plasma responses at angular frequencies $\omega_0 - \omega_1$ and $\omega_0 + \omega_1$. In the case where the frequency difference (beat-wave frequency) plays a dominant role in the plasma response (Stokes coupling), the process is referred to as a beat-wave plasma interaction. In the up-conversion interaction, the frequency sum is the dominant plasma response (anti-Stokes coupling). The dominance of a beat-wave interaction can be arranged by choosing a proper interaction geometry, frequency, and wave number to match plasma modes and the appropriate intensities of the driver pumps. The strongest nonlinear coupling to the plasma takes place if the beat-wave frequency $\Omega = \omega_0 - \omega_1$ and the beat-wave vector satisfy the following selection rules:

$$n\Omega = \omega_H + \omega_L \quad (2a)$$

$$n(\mathbf{k}_0 - \mathbf{k}_1) = \mathbf{k}_L + \mathbf{k}_H \quad (2b)$$

(where $n = 1, 2, \ldots$). Here (ω_L, $\mathbf{k}_L$) and (ω_H, $\mathbf{k}_H$) are the angular frequencies and wave numbers of low (L)- and high (H)-frequency plasma eigenmodes, respectively.

The early phase of a beat-wave plasma interaction is fully described by the corresponding nonlinear (with respect to E_0 and E_1) dispersion relation. The nonlinear mode-mode coupling dispersion relation in the lowest-order coupling approximation, $n = 1$, can be written as (*19–21*) (Stokes coupling):

$$\epsilon_L(\omega_L,\mathbf{k}_L)\epsilon_H[\omega_L - \Omega,\mathbf{k}_L - (\mathbf{k}_0 - \mathbf{k}_1)] = \mu(\mathbf{E}_0^2,\mathbf{E}_1^2,\Omega);\ |\mu| \ll 1 \quad (3)$$

In Eq. 3 ϵ_L,ϵ_H are dielectric permittivities of the low- and high-frequency plasma eigenmodes and μ is the nonlinear coupling coefficient which defines the threshold for the interaction. A variety of nonlinear processes can be described by Eq. 3, such as Stokes sideband coupling and stimulated Raman and Brillouin scattering (*20–22*).

In the case where the beat-wave frequency, or its harmonic, resonates directly with the frequency of a plasma eigenmode

$$n\Omega = \omega_L \quad (4a)$$

$$n(\mathbf{k}_0 - \mathbf{k}_1) = \mathbf{k}_L \quad (4b)$$

(where $n = 1, 2, \ldots$), a strong nonlinear interaction can occur that we will refer to as nonlinear mixing (*1, 23, 24*). A coupled-mode equation describing the low-frequency plasma response induced by nonlinear mixing can be cast in the form (*14, 15, 25*)

$$\epsilon_L\left[\omega + i\frac{\partial}{\partial t},\mathbf{k}_L - i\nabla\right] a_L = \beta(\mathbf{E}_0 \cdot \mathbf{E}_1, \ldots);\ |\beta| \ll 1 \quad (5)$$

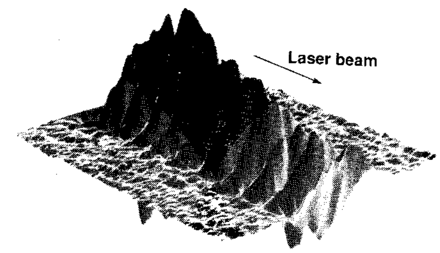
for the lowest order coupling, $n = 1$, where β is a nonlinear coupling coefficient that is bilinear in the amplitudes of the two externally applied pump waves and a_L is the amplitude of the low-frequency plasma eigenmode. Nonlinear mixing is not subject to the same kind of threshold condition as in Eq. 3 and produces a finite-amplitude beat-wave response for arbitrarily small-amplitude driver pumps (*23, 24*). To distinguish between cases described by Eqs. 3 and 5, we will refer to Eq. 3 as parametric mixing (*26, 27*).

Beat-Wave Plasma Probing, Heating, and Ionospheric Plasma Modification

Plasma density probing by two crossed laser beams with good space-time resolution and without significant disturbance of plasma was proposed by Kroll *et al.* (*1*). In this paper it was assumed that the beat frequency is equal to the electron plasma frequency (ω_{pe}) so that the Langmuir plasma modes can be resonantly excited. The plasma density can be measured in two ways: (i) by observing the scattering of two beams at $\omega_1 - \omega_{pe}$ (or $\omega_1 + 2\omega_{pe}$) or (ii) by measuring the scattering of the third (probing) laser beam by the nonlinearly excited Langmuir modes. The theoretical prediction of Kroll *et al.* showed that the ratio of the scattered to incident energy flux was about 10^{-7}. This result gave encouragement to the use of beat waves in plasma diagnostics (*2*). The measurement of the local electron density and its fluctuations in a cold magnetized plasma resulting from nonlinear mixing of the electromagnetic waves was addressed by Etievant *et al.* (*28*). In this research two incident electromagnetic waves propagated perpendicular to the magnetic field and excited either an extraordinary or an ordinary wave, which was used as a diagnostic tool. Weyl (*29*) studied excitation of electrostatic oscillations (upper hybrid modes) by means of two ruby laser beams interacting with the plasma in a Q machine (a device that generates a low-temperature plasma). In this case the excited upper hybrid mode is detected with a third laser beam scattered by the plasma. In both cases (excitation of electromagnetic or electrostatic plasma modes), the methods have the inherent advantage of not requiring a probe to be put in plasma, avoiding in that way significant plasma disturbances induced by the probes. Furthermore, in the case of the excitation of plasma modes by nonlinear mixing of electromagnetic waves, one can control the amplitude and the direction of the wave vector of the excited plasma modes by changing the orientation of the interacting electromagnetic beams.

Mixing of electromagnetic waves also can be applied in plasma heating. Magnetically confined plasma heating due to the nonlinear mixing of electromagnetic waves was studied by Thompson and collaborators (*3, 4*). In the first paper (*3*), two incident waves were of such high frequency that the plasma did not affect their propagation. The beat frequency was chosen to be equal to the ion-cyclotron frequency so that ions were heated directly. It was suggested that diffuse plasma be first heated to near thermonuclear temperatures and then compressed to a high density. Somewhat later (*4*), incident waves were selected to be plasma waves (extraordinary plasma modes) at much lower frequency. In this case the input power was increased and there was accordingly more energy absorption as compared to the total transparency case (*3*). Heating of magnetically confined plasma in the electron-cyclotron frequency range as a result of the parametric mixing was addressed by Stefan (*30*), who showed that by choosing a proper beat frequency a significant deposition of the external energy can take place in the low-density regions of tokamaks. Heating of inertially confined plasma by parametric mixing of laser radiation was addressed by several workers (*6, 7, 19*). Using the nonresonant double-driver pump parametric theory developed by Aliev and Zunder (*6*) [generalization of the theory to include dc magnetic field is given by Stefan (*19*)], Kruer showed (*7*) that a significant deposition of laser beat energy takes place in subcritical laser plasma as a result of the nonresonant parametric excitation of Langmuir modes coupled to ion-acoustic waves.

Fig. 1. Three-dimensional plot of the plasma wave electric field excited by means of beating laser beams ($\omega_0 = 5\omega_{pe}$, $\omega_1 = 4\omega_{pe}$). The electric field reaches a maximum value $\varepsilon = 0.5$ at the left boundary. [Adapted from (*37*) with permission of *Physical Review Letters*, copyright 1985]



Nonlinear mixing was discussed by Rosenbluth and Liu (*5*), Cohen *et al.* (*25*), and recently by Kruer and Estabrook (*8*) who reported that beat-wave heating of a laser plasma need not be a significant source of suprathermal electrons. Rosenbluth and Liu (*5*) generalized the theory of nonlinear mixing (*1*), including nonlinearities in the large-amplitude plasma wave and plasma inhomogeneities. Cohen *et al.* (*25*) showed that plasma can be heated if $\Omega = \omega_{pe}$ by nonlinear damping of Langmuir modes due to the cascading process. The process is optimized if the laser beams are parallel, laser intensities are roughly equal, and the frequency mismatch $\Delta = \Omega - \omega_{pe}$ is less than or equal to the linear damping rate of the Langmuir modes.

A beat-wave interaction with ionospheric plasma [called double-resonance excitation (*9, 10*)] was experimentally studied by Wong *et al.* (*31*). Two electromagnetic waves with a beat frequency of the order of ionic frequencies are radiated toward the ionosphere. In this way, ion-acoustic and electrostatic ion-cyclotron waves were excited. The low-frequency (LF) ionospheric modes were studied by detection of the radar waves that were Thomson-scattered from the LF perturbation. They used a relatively small, ground-based antenna for excitation because of the lower threshold for parametric coupling (*32, 33*).

Plasma instabilities with multiple driver pumps were observed in the Arecibo (Puerto Rico) experiment (*34*) with 430-MHz incoherent backscatter radar. The beat frequency ranged from 0 to 7 kHz. At zero beat frequency, standard decay and purely growing-mode instabilities were observed. At a finite beat frequency, the decay instability usually disappeared and was replaced by arithmetic mean $(\omega_0 + \omega_1)/2$ instability (*10*), corresponding to the two-plasmon parametric instability of ionospheric plasma in the presence of the beat driver pump. A very efficient parametric excitation of long- and short-wavelength Bernstein modes by an X-mode driver pump in the ionosphere was theoretically predicted by Stefan (*19*).

Many methods for probing the ionospheric plasma are difficult and costly, in contrast to the beat-wave interaction, which can provide a reliable and low-cost method for in situ and remote-sensing measurements. Although radar beams are usually used in contemporary experiments, tunable laser beams (*35*) show a number of advantages and very likely will play a major role in future ionospheric modification research.

Beat-Wave Accelerators

Two beat-wave particle accelerator schemes are under serious investigation at present: the plasma beat-wave accelerator (PBWA) and the inverse free electron laser (IFEL) accelerator. In the PBWA, the beating pattern between two copropagating laser beams of slightly different frequencies is used to organize plasma electrons into a space-charge density wave, which in turn accelerates injected particles (*11*). In the IFEL, the beating pattern between a light wave and a static wiggler field is used to accelerate particles directly (*12*).

The promise of ultrahigh accelerating gradients is the main attraction of the PBWA. The maximum electric field of a space-charge density wave propagating close to the speed of light is approximately equal to the square root of the plasma density per cubic centimeter. Thus, for instance, the accelerating electric field of such a relativistic plasma wave with a background density of 10^{16} cm^{-3} can be as high as 10^4 MV/m compared to the typical gradient of 20 MV/m in present-day machines. In a uniform plasma the plasma wave can be thought of as a single-mode, slow-wave structure in which the wavelength of the accelerating wave is typically a few hundred micrometers instead of a few centimeters as in linear accelerators. This ultrahigh-gradient, ultrashort-wavelength regime represents an extremum of the parameter space for designing particle accelerators with available power sources (lasers).

In the PBWA, the beat-wave frequency Ω of the two copropagating laser beams is chosen to exactly match the plasma frequency ω_p. The ponderomotive force F_{NL}, due to the gradient of the intensity of the amplitude-modulated beat-wave pattern, then resonantly builds up a plasma wave (Fig. 1). Physically, the plasma wave consists of regions of space charge, which propagate with a phase velocity v_{ph} that is equal to the group velocity of the light waves $v_g = c(1 - \omega_p^2/\omega_0^2)^{1/2}$ (where c is the speed of light). If an electron is now injected with a velocity close to this, it can be trapped by the plasma wave and gain energy from it in much the same way as a surfer riding an ocean wave.

The amount of energy gained by an electron as it is acclerated by such a wave depends on the amplitude and speed of the wave. If we define the relativistic Lorentz factor

$$\gamma_{ph} = (1 - v_{ph}^2/c^2)^{-1/2} \cong \omega_0/\omega_p$$

it can be shown (*13*) that the maximum energy gain is limited [because electrons eventually outrun the accelerating electric field (dephasing)] to $\Delta W = \epsilon\gamma_{ph}^2 m_e c^2$ (m_e is the mass of the electron). Here ϵ is the fractional density perturbation n_1/n_0 associated with the plasma wave. For instance, $\epsilon = 0.2$ and $\gamma_{ph} = 100$ would give $\Delta W =$ 1 GeV. If a 1-μm laser is used, $\gamma_{ph} = 100$ implies a plasma density of 10^{17} cm^{-3} and an accelerating gradient of 6.3 GeV/m.

The amount of laser power needed to excite such a large plasma wave has been calculated (*5*) and confirmed in two-dimensional particle simulations (*36*). Because there are numerous other instabilities in a laser-excited plasma (*37, 38*), an extremely short (few-picosecond) laser pulse containing the two frequencies must be used in the PBWA.

An important issue in the PBWA is the overall efficiency η. This is the product of three efficiencies: (i) the ratio of wall plug power to laser power, (ii) conversion of laser energy to plasma wave energy, and (iii) conversion of plasma wave energy into that of accelerated particles. The laser efficiency could in principle be as high as 10% for a short-pulse, high repetition rate CO_2 or excimer laser if multiplexing techniques are used. The laser to plasma wave coupling efficiency may appear at first glance to be limited by the conservation of wave action to ω_p/ω_0. However, this is not so because nonlinear mixing generates electromagnetic sidebands at Stokes and anti-Stokes frequencies, $\omega_o - n\omega_p$, $\omega_0 + n\omega_p$. By choosing an appropriate initial frequency mismatch between Ω and ω_p, one can preferentially couple to Stokes sidebands and thereby improve this coupling efficiency (*25, 39*). Thus, for $\gamma_{ph} = 100$ we may expect the laser to plasma wave coupling efficiency to be 10% rather than 1% as expected from ω_p/ω_0. The beam loading efficiency (plasma wave to accelerated particles) must be consistent with the emittance and energy spread requirements imposed by the end use. To reduce the energy spread without lowering the beam loading efficiency, van der Meer has suggested the use of an accelerating electron bunch that is ramped down in density (*40*). In the two-dimensional analysis, beam loading efficiencies of up to 20% appear feasible (*41*). Thus the

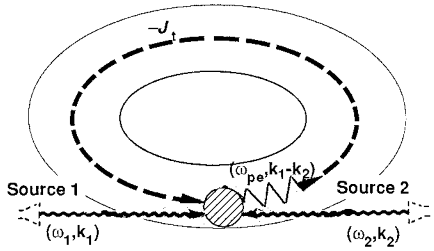


Fig. 2. Current drive in tokamaks based on the use of two high-frequency electromagnetic pump waves $(\omega_1,\mathbf{k}_0)$, $(\omega_1,\mathbf{k}_1)$ which readily propagate through plasma. The excited beat wave can be a longitudinal, well-defined mode $(\omega_{pe},\mathbf{k}_0 - \mathbf{k}_1)$ or a quasi-mode that couples well to the plasma and drives current efficiently.

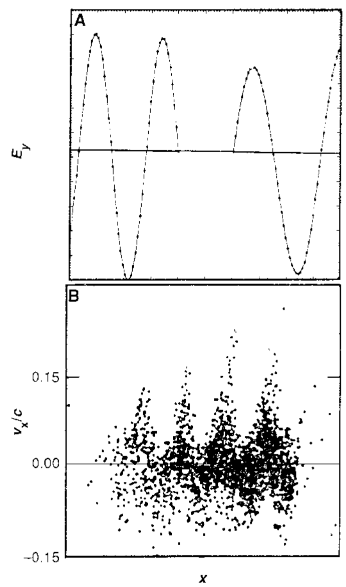


Fig. 3. Beat-wave heating in a finite, inhomogeneous medium: (**A**) the right- and left-going electromagnetic waves before the onset of beating; (**B**) the (x,v_x) phase space after a fairly large-amplitude electron plasma wave has been established. [Adapted from (*52*) with permission of *Physics of Fluids*, copyright 1975]

overall efficiency is likely to be about 2×10^{-3} for the PBWA.

Experiments are under way (*42*) at the University of California, Los Angeles (UCLA), Rutherford Laboratory (United Kingdom), Institute for Laser Engineering (Osaka, Japan), Institut de la Recherche Scientifique (Quebec, Canada), and elsewhere to demonstrate the excitation of the relativistic plasma wave by the laser beat wave in a reproducible fashion and to demonstrate controlled acceleration of injected test particles. In a recent UCLA experiment the relativistic plasma wave was excited by beating the 9.6-μm and 10.6-μm lines of a CO_2 laser, with a modest intensity of 2×10^{13} W/cm^2 in a density plasma of 10^{17}/cm^3 (*43*). The plasma wave electric field was inferred from the Thomson scattering of a probe laser beam to be 10^3 MeV/m, a substantial improvement over the present benchmark gradient for accelerators. A new mechanism, which saturates the beat-excited plasma wave in this parameter regime, was discovered (*44*). The relativistic plasma wave saturates, on the time scale of a few picoseconds, by coupling to other plasma modes that have a much lower phase velocity, via an ion ripple due to stimulated Brillouin scattering of the laser beams. A scaled-up experiment designed to demonstrate controlled acceleration of injected electrons is currently under way at UCLA.

In contrast to the PBWA, the IFEL acceleration mechanism is inherently simple in that it does not rely on collective effects. It works as follows. The combined action of the laser $(\omega_0,\mathbf{k}_0)$ and a static wiggler field $(0,\mathbf{k}_w)$ on a relativistic electron bunch results in a ponderomotive wave, which has a phase velocity slightly less than c because

$$v_{ph} = \omega_0/(k_0 + k_w) = c/(1 + k_w/k_0)$$

Thus, the ponderomotive well can trap and continuously accelerate electrons, provided that the resonance condition that relates the electron energy to the wiggler and laser parameters is maintained. This is given by

$$\gamma = (1 + a_w)^{1/2} (\lambda_w/2\lambda_0)^{1/2}$$

where λ is wavelength and $a_w = eA_w/mc^2$ is the normalized vector potential (A_w is the unnormalized vector potential). As γ increases, either a_w or λ_w or both, must increase. Unfortunately a_w cannot be increased too much because in an IFEL the electron beam loses energy to synchrotron radiation as it is continuously being bent. In fact, synchrotron loss eventually will limit the maximum energy gain in an IFEL to a few hundred gigaelectron volts. The main attraction of the IFEL is that the laser, injector, and wiggler technology exists for building a prototype device to a gigaelectron volt level.

Beat-Wave Current Drive in Tokamaks

In addition to the numerous applications of beat-wave scattering to plasma diagnostics, plasma heating, ionospheric modification, and advanced acceleration schemes, the beating of two intense microwave sources can be used to drive current in toroidal devices (*14, 15*). In an effort to extend the lifetime of a tokamak discharge with the ultimate goal of achieving steady-state operation, a number of noninductive, current-drive schemes have been analyzed theoretically and studied experimentally, for example, injection of neutral beams, compact tori, and electromagnetic waves (*46, 47*).

Beat-wave scattering can provide an alternative current-drive mechanism that has excellent penetration characteristics and for which there is control over both the population of resonant particles carrying the current and the location of current deposition in the torus. Such a current-drive scheme could both sustain the plasma and control the current profile so as to stabilize the plasma with respect to magnetohydrodynamic instabilities. Furthermore, the required electromagnetic waves are easily injected through small ports in the wall of the torus by quasi-optical means.

In beat-wave current drive, two electromagnetic waves are introduced at finite amplitude. When their beat-wave frequency, $\omega_0 - \omega_1$, and the corresponding beat-wave number, $\mathbf{k}_0 - \mathbf{k}_1$, resonate with a plasma mode, a large-amplitude beat wave is locally excited in the plasma. The electromagnetic pump waves are high-frequency waves and are readily propagated through the plasma. The excited beat wave can be a longitudinal wave or a quasi-mode that couples well to the plasma and drives current efficiently (Figs. 2 and 3). Here we assume that the ratio of ω_0 to the beat frequency is not large, so that dispersion and other dephasing effects (for example, inhomogeneity) effectively prevent induced electromagnetic sidebands and upward or downward cascading (*25, 39*).

The beat waves are nonlinearly excited in response to the ponderomotive force arising from the electromagnetic forces on a charged particle that are bilinear in the wave amplitudes of the two electromagnetic pump waves (*14, 15, 48, 49*). The plasma perturbation associated with the beat wave, in turn, alters the propagation of each electromagnetic wave by contributing to nonlinear currents that are bilinear in the amplitudes of the beat wave and the other electromagnetic wave. Analytical self-consistent solution of plasma equations of motion and Maxwell's equations, with the use of conventional perturbation theory (*14, 15, 48*), leads to coupled-mode equations describing the beat-wave scattering of transverse waves in a magnetized plasma. From these coupled-mode equations, it was rigorously demonstrated that the total wave action density in the transverse waves is conserved (*14, 15, 48*). The wave energies are given by the product of the wave frequencies and the respective actions, and the wave momenta are given by the product of the wave numbers and the respective actions. As a consequence of the phase-matching conditions for beat-wave scattering, $\omega_0 = \omega_1 + \omega_2$, $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$, and the total wave action density is conserved; for every photon of the high-frequency pump wave that decays, an additional photon at the lower frequency wave appears whose energy is a fraction ω_1/ω_0 smaller than the original photon and whose momentum is k_1 per photon rather than k_0 per photon. This is a statement of the Manley-Rowe relations (*50*). Because energy and momentum are also conserved, there are upper bounds on the energy and momentum available to the beat wave and the plasma.

In terms of the wave-action transfer ΔJ, the beat wave acquires $\omega_2 \Delta J$ of energy and $k_2 \Delta J$ of momentum. Damping of the beat wave by resonant electrons makes this energy and momentum available to electrons that can carry current. Unless additional absorption or scattering processes occur, $\omega_2 \Delta J$ and $k_2 \Delta J$ are the upper limits on the energy and momentum that can be deposited in the plasma. The amount of current deposited in the plasma follows directly from the momentum transfer and knowledge of the velocity of the resonant electrons. Relative to the transverse wave energy introduced into the plasma, $\omega_0 J_0^{\text{in}} + \omega_1 J_1^{\text{in}}$, the fraction of energy available to the beat wave and the plasma is

$$\frac{\omega_2 \Delta J}{\omega_0^{\text{in}} J_0^{\text{in}} + \omega_1 J_1^{\text{in}}} \leq \frac{\omega_2 J_0^{\text{in}}}{\omega_0 J_0^{\text{in}} + \omega_1 J_1^{\text{in}}} \equiv q_e$$

because $\Delta J \leq J_0^{\text{in}}$ in consequence of action conservation and $q_e \leq 1$ is the so-called quantum efficiency. The net current-drive efficiency for beat-wave current drive is reduced from the ideal current-drive efficiency of the particular beat wave (if it was directly launched) by the product $q_e \Delta J/J_0^{\text{in}}$.

The amount of action transfer realized depends nonlinearly on the wave intensities. For small-amplitude transverse waves, plasma inhomogeneity is a dominant mechanism for saturating the beat-wave scattering (*15, 25, 48*). The eikonal (geometric optics) solution of the coupled mode equations for steady-state transfer of action flux across a nonuniform plasma with linear gradients when the transverse pump waves are directed parallel or antiparallel indicates that $\Delta J/J_0^{\text{in}} \rightarrow 1$ when

$$(k_2^2/k_0 k_1)\pi k_0 L \,|\, v_0 v_1/c^2 \,| > 1$$

where L is the characteristic gradient length for the plasma dielectric response to the beat wave, and v_0 and v_1 are the magnitudes of the linear oscillation velocities of electrons in the transverse waves. The wave action transfer is a monotonically increasing function of the product of the wave intensities (*14, 15, 45, 48*). However, the transverse wave intensities are limited not only by technology but also by the increased possibility of stimulated scattering parametric instabilities (*21, 37, 38, 49*) that could steal photons away at higher wave amplitudes. Furthermore, because of action conservation, only the energy and wave action in the higher frequency pump wave are available to the beat wave and the plasma unless additional absorption and scattering occur (*25, 39*). Thus, most of the available transverse wave energy should be injected in the higher frequency wave subject to maintaining the product of $|v_0 v_1|$ at some satisfactory level.

Having excited a large-amplitude longitudinal wave in the plasma, we want to dissipate this wave so as to create a population of current-carrying particles that has good characteristics for sustaining a toroidal current on a confinement time scale. Various candidates for the best plasma wave or quasi-mode for this purpose are being studied. There is considerable flexibility in the type of beat wave that can be launched by adjusting the frequencies and the spatial orientations of the two transverse pump waves. Detailed calculations and numerical simulations have so far considered Langmuir beat waves propagating nearly parallel to the magnetic field, which is just the limiting case of a lower hybrid wave when the electron cyclotron frequency exceeds the electron plasma frequency. Right and left circularly polarized electron cyclotron waves introduced nearly tangent to the toroidal magnetic field are appropriate transverse pump waves.

For a Langmuir beat wave propagating parallel to the magnetic field, the resonant electrons have velocities equal to the beat-wave phase velocity. As this velocity is increased, the resonant electrons are more energetic and much less collisional. The current-drive efficiency improves with increasing particle energy until relativistic effects cause saturation (*46, 51*) and there are insufficient numbers of energetic electrons to adequately damp the beat wave. Thus, there is an optimum range of phase velocities for efficient current drive.

Numerical simulations have been conducted with a self-consistent, electromagnetic, relativistic, particle code EMONE (*52*) to study the beat-wave coupling to Langmuir waves, the damping of the Langmuir waves by resonant electrons, and the role of ions. A novel diagnostic calculates the integrated current as the electron velocity distribution collisionally relaxes after it has been perturbed by a short pulse of transverse waves (*53*). The simulations have revealed the role of parasitic stimulated scattering instabilities and the role of ions in competing for the beat-wave momentum. The results indicate that virtually complete action transfer and good coupling of the beat waves to the electrons can be achieved for beat-wave phase velocities less than $4(T_e/m_e)^{1/2}$, where T is temperature. In the simulations, current-drive efficiencies $n_e I R_0/P \leq 0.8 \times 10^{20}$ A m^{-2} W^{-1} have been attained, where n_e is the electron density $= 10^{20}$ m^{-3}, R_0 is the major radius in meters, I is the current, and P is the power. The best lower hybrid current-drive efficiencies reported in experiments have not exceeded 0.3×10^{20} A m^{-2} W^{-1} (*46*).

The significant intensity and frequency requirements on the transverse wave sources appropriate to sustaining currents in large tokamaks of interest today are met in the free-electron-laser microwave source (*54*) being built for the Microwave Tokamak Experiment (MTX) (*55*) at Lawrence Livermore National Laboratory with the cooperation of the Massachusetts Institute of Technology. Realistic scenarios have been calculated for proposed current-drive experiments in MTX and in a design study of an engineering test reactor tokamak (*15*).

Beat Wave–Driven Free Electron Laser

Two copropagating laser beams $(\omega_0, \mathbf{k}_0)$, $(\omega_1, \mathbf{k}_1)$ in antiparallel interaction with a relativistic electron beam (REB) can be also used to induce a laser process. This would have applications to laser fusion (*5–7*), ground-based FELs (*56*), and space science. The process is based on stimulated Raman backscattering of a beat laser driver on Doppler-shifted REB eigenmodes. The frequency of the scattered mode in the laboratory reference frame is given by $\omega_s = 4\gamma^2(\omega_0 - \omega_1)$, where γ is the relativistic Lorentz factor. The threshold (*26, 27*) for the scattering process in the beam reference frame depends on $(2\gamma\Omega - \omega_B')^{1/2}/(\omega_B')^{1/2}$, where ω_B' is the frequency of the REB eigenmode in a beam system.

In order to obtain a higher laser frequency and accordingly shorter wavelengths with good coupling efficiency, a magnetized REB can be used. In this case, one can use the harmonics of electron-Bernstein modes as scattering modes by choosing $\Omega = n\Omega_e$, $n = 1, 2, \cdots$, where Ω_e is the electron-cyclotron frequency in the beam system (*16*). The threshold for scattering is lower in the case of a beat-wave pump than in the case of a single driver (*16*). For $n = 10$ and $\gamma = 10$, FEL wavelengths $\lambda_{\text{FEL}} = 0.8$ to 1.6 μm (*56*) can be easily obtained if the guiding longitudinal magnetic field is of the order of 10 kG. The efficiency of this laser can be significantly higher than that of a FEL with a conventional wiggler field (*57*). Efficient generation of millimeter radiation can also be accomplished by a beat-wave FEL and used for heating magnetically confined plasmas at electron-cyclotron resonance or for diagnostics of high-β plasma ($\beta = 8\pi n\kappa T/B^2$, where n is the plasma density, κ is Boltzmann's constant, and B is the confining magnetic field in cgs units).

Phase Conjugation with Beat Waves

Lastly, we describe qualitatively the generation of phase conjugate waves with the use of the beat technique (*18*). It is well known from laser-plasma interaction experiments that parametric three-wave interactions, such as stimulated Brillouin and Raman scattering, generate scattered electromagnetic waves that exhibit some of the properties of a truly phase conjugate wave (*58*). The beat-wave acceleration mechanism can be thought of as a nondegenerate, four-wave collinear optical mixing process that is analogous to the Raman forward-scattering instability (*59*). In "phase-conjugation language" the pump and the probe electromagnetic waves can be made to beat at any arbitrary angle, whereupon k-matching (when all the vectors add up) determines that a second counterpropagating pump wave scatters light by Bragg diffraction from a plasma grating in the direction opposite to that of the probe, giving rise to a phase conjugate wave (*60*). The beating of the pump and the probe can be carried out at either the ion-acoustic frequency (*61, 62*) or the Bohm-Gross frequency (*63*). In the former case the plasma grating is an ion-acoustic wave, whereas in the latter case the grating is an electron-plasma wave whose periodicity depends on the angles between the two pumps and the probe waves.

It is possible to obtain phase conjugation by degenerate four-wave mixing (DFWM) in a plasma (*64, 65*). In this case the beating of the pump and the probe waves gives rise to a stationary density perturbation that can scatter the second counterpropagating pump wave in much in the same way as an ordinary grating produces a diffracted order. Phase conjugation by DFWM in a plasma has applications in the areas of pointing and tracking, coherent image amplification and communications, particularly at microwave frequencies. This is so because the third-order susceptibility $\chi^{(3)}$, responsible for DFWM in a plasma, is proportional to n_e/n_c^2, where n_e is the plasma density and n_c is the critical density for $\omega_0 = \omega_{pe}$. Thus in the microwave and far-infrared regions $\chi^{(3)}$ of a plasma can be orders of magnitude larger than those for other currently available materials.

Conclusions

The beat-wave plasma interaction is an important and rapidly growing research area in plasma physics and plasma technology. We have described an exciting recent beat-wave application to driving current in tokamak plasmas with the use of new intense FEL microwave sources. This current-drive scheme has a number of technical advantages, for example, good penetration to the magnetic axis by waves that are easily launched externally through small ports, and it offers the possibility of both sustaining the toroidal current and controlling the current profile. Analysis and self-consistent particle simulations have demonstrated that beat-wave current drive can lead to attractive efficiencies in the MTX experiment and in reactor-grade tokamaks. Beat waves have also been proposed as a means to heat magnetically and inertially confined plasmas. By means of beat waves, control over the location of energy deposition, density profile, and the generation of suprathermal particles is enhanced. The probing of ionospheric plasma with beat waves appears to provide a low-cost and reliable method for in situ and remote-sensing measurements. Furthermore, phase conjugation by four-wave mixing in a plasma opens up the possibility of coherent image amplification. In addition, modulated laser beat waves will very likely be important in future upper atmospheric research. Beat waves might also find applications in a wiggler-free FEL producing submicrometer wavelengths with higher efficiency. Particle acceleration with beat waves may lead to a new class of ultrahigh-gradient, compact accelerators for use in science and industry.

REFERENCES AND NOTES

1. N. M. Kroll, A. Ron, N. Rostoker, *Phys. Rev. Lett.* **13**, 83 (1964).
2. B. L. Stansfield, R. Nodwell, J. Meyer, *ibid.* **26**, 1219 (1971).
3. C. R. James and W. B. Thompson, *Can. J. Phys.* **45**, 1771 (1967).
4. P. Jayasimha and W. B. Thompson, *ibid.* **48**, 1894 (1970).
5. M. N. Rosenbluth and C. S. Liu, *Phys. Rev. Lett.* **29**, 701 (1972).
6. Yu. M. Aliev and D. Zunder, *Sov. Phys. JETP* **61**, 1057 (1971).
7. W. L. Kruer, *Phys. Fluids* **16**, 1548 (1973).
8. ______ and K. Estabrook, *ibid.* **29**, 3493 (1986).
9. D. Arnush, K. Nishikawa, B. D. Fried, C. F. Kennel, A. Y. Wong, *ibid.* **16**, 2270 (1973); B. D. Fried, A. Adler, R. Bingham, *J. Plasma Phys.* **24**, 315 (1980).
10. J. A. Fejer, B. L. Cragin, R. L. Showen, *J. Plasma Phys.* **19**, 355 (1978).
11. T. Tajima and J. M. Dawson, *Phys. Rev. Lett.* **43**, 267 (1979).
12. C. Pellegrini, in *Laser Acceleration of Particles*, AIP Conference Proceedings No. 91, P. Channell, Ed. (American Institute of Physics, New York, 1982), pp. 138–149.
13. C. Joshi, T. Tajima, J. M. Dawson, H. Baldis, N. A. Ebrahim, *Phys. Rev. Lett.* **47**, 1285 (1981).
14. B. I. Cohen, *Comments Plasma Phys. Controlled Fusion* **8**, 197 (1984).
15. B. I. Cohen *et al.*, *Nucl. Fusion* **28**, 1519 (1988).
16. V. Stefan, D. E. Parks, M. Rotenberg, E. M. Waisman, A. R. Wilson, *Bull. Am. Phys. Soc.* **32**, 1713 (1987).
17. V. Stefan and A. R. Wilson, S-Cubed internal material SSS-P-87-8520259 (also to appear as part of the Symposium on Innovative Science and Technology for Government and Civilian Applications, Society of Photo-Optical Instrumentation Engineers, OE/Lase 89, Los Angeles, 15 to 20 January 1989).
18. D. Steel and J. F. Lam, *Opt. Lett.* **4**, 363 (1974).
19. V. Stefan, *Phys. Fluids* **26**, 1789 (1983); *ibid.*, p. 1797.
20. B. I. Cohen, *ibid.* **30**, 2676 (1987).
21. V. Stefan, N. A. Krall, J. B. McBride, *ibid.*, p. 3703.
22. C. J. McKinstrie and R. Bingham, "The modulation instability of nonlinearly coupled waves" (Report LA-UR88-0459, Los Alamos National Laboratory, Los Alamos, NM, March 1988).
23. V. N. Tsytovich and A. B. Shvartsburg, *Sov. Phys. Tech. Phys.* **11**, 1431 (1967).
24. L. M. Kovrizhnykh, *ibid.*, p. 1183.
25. B. Cohen, A. Kaufman, K. M. Watson, *Phys. Rev. Lett.* **29**, 581 (1972).
26. V. P. Silin, *Parametric Action of High Power Radiation on a Plasma* (Nauka, Moscow, 1973) (in Russian).
27. P. K. Kaw, W. L. Kruer, C. S. Liu, K. Nishikawa, *Adv. Plasma Phys.* **6**, 3 (1976).
28. C. Etievant, S. Ossakaw, E. Ozizmir, C. H. Su, *Phys. Fluids* **11**, 1778 (1968).
29. G. Weyl, *ibid.* **13**, 1802 (1970).
30. V. Stefan, in *Heating in Toriodal Plasmas*, Proceedings of the 2nd Joint Grenoble-Varenna International Symposium on Heating in Toroidal Plasmas, Como, 1980 (Commission of the European Communities, Directorate-General XII, Fusion Programme, Brussels, Belgium, 1980), vol. 2, pp. 1149–1150.
31. A. Y. Wong, J. Santoru, G. G. Sivjee, *J. Geophys. Res.* **86**, 7718 (1981).
32. R. L. Stenzel and A. Y. Wong, *Phys. Rev. Lett.* **28**, 274 (1972).
33. A. Y. Wong and R. J. Taylor, *ibid.* **27**, 644 (1971).
34. R. Showen, L. M. Duncan, B. L. Cragin, *Geophys. Res. Lett.* **5**, 187 (1978).
35. K. A. Brueckner and V. Stefan, La Jolla Institute, internal material LJI-LJ-88-272P (1988) (unpublished); V. Stefan, Institute for Nonlinear Science, University of California, San Diego, internal material UCSD-INLS-88-6535 (1988) (unpublished).
36. C. Joshi *et al.*, *Nature* **311**, 25 (1984).
37. D. W. Forslund, J. M. Kindel, W. B. Mori, C. Joshi, J. M. Dawson, *Phys. Rev. Lett.* **54**, 558 (1985).
38. W. B. Mori *et al.*, *ibid.* **60**, 1298 (1988).
39. S. J. Karttunen and R. R. E. Salomaa, *ibid.* **56**, 604 (1986).
40. S. van der Meer, *CERN Rep. PS/85-65* (1985).
41. S. Wilks, T. Katsouleas, J. M. Dawson, P. Chen, J. J. Su, *IEEE Trans. Plasma Sci.* **PS-15**, 210 (1987).
42. See, for instance, *New Developments in Particle Acceleration Techniques*, S. Turner, Ed. (CERN, Geneva, 1987); A. M. Sessler, *Phys. Today* (January 1988), p. 26.
43. C. Clayton, C. Joshi, C. Darrow, D. Umstadter, *Phys. Rev. Lett.* **54**, 2343 (1985).
44. C. Darrow *et al.*, *ibid.* **56**, 2629 (1986).
45. R. R. E. Salomaa and S. J. Karttunen, *Phys. Scr.* **33**, 370 (1986).
46. N. J. Fisch, *Rev. Mod. Phys.* **59**, 175 (1987) and references therein.
47. A. Bers, U.S. Patent 4,292,125 (29 September 1981).
48. A. N. Kaufman and B. I. Cohen, *Phys. Rev. Lett.* **30**, 1306 (1973).
49. J. F. Drake *et al.*, *Phys. Fluids* **17**, 778 (1974).
50. W. H. Louisell, *Coupled Mode and Parametric Electronics* (Wiley, New York, 1960); R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, New York, 1969).

51. N. J. Fisch, *Phys. Rev. A* **14**, 3245 (1981).
52. B. I. Cohen *et al.*, *Phys. Fluids* **18**, 470 (1975).
53. R. H. Cohen, *ibid.* **30**, 2442 (1987); *Bull. Am. Phys. Soc.* **32**, 1791 (1987).
54. T. J. Orzechowski *et al.*, *Phys. Rev. Lett.* **57**, 2172 (1986).
55. K. I. Thomassen *et al.*, *Free-Electron-Laser Experiments in Alcator-C* (Government Printing Office, Washington, DC, 1986); K. I. Thomassen *et al.*, *Bull. Am. Phys. Soc.* **31**, 1515 (1986).
56. B. G. Levi, *Phys. Today* (June 1987), p. 17.
57. T. C. Marshall, *Free-Electron Lasers* (Macmillan, New York, 1985), chap. 4.
58. R. H. Lehmberg, *Phys. Rev. Lett.* **41**, 863 (1978).
59. C. Joshi *et al.*, *ibid.* **47**, 1285 (1981).
60. D. M. Pepper, in *Laser Handbook*, M. L. Stitch and E. D. Schulz-Dubois, Eds. (North-Holland, Dordrecht, 1985), vol. 4, p. 333.
61. C. Pawley *et al.*, *Phys. Rev. Lett.* **49**, 877 (1983).
62. I. Nebenzahl, A. Ron. N. Rostoker, *ibid.* **60**, 1030 (1988).
63. B. Amini and F. F. Chen, *ibid.* **53**, 1441 (1984).
64. J. F. Federici and D. K. Mansfield, *J. Opt. Soc. Am. B* **3**, 1588 (1986).
65. Y. Kitagawa *et al.*, in preparation.
66. This work was supported by Department of Energy contracts DE-AS03-83-ER40120 and DE-F603-87ER13752 at UCLA and W-7405-Eng-48 at Lawrence Livermore National Laboratory, and by S-Cubed, Institute for Nonlinear Science (University of California, San Diego), and La Jolla Institute internal funds.