Unified theory of parametric excitations in magnetized plasma produced by the action of nonmonochromatic driver pump. I. Modulated driver pump

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The theory of parametric excitations in a magnetized plasma by the action of a driver pump with time-varying amplitude and phase is developed. As a concrete example, amplitude modulation in the form of rectangular pulse train and phase modulation in a sinusoidal form are assumed. Both nonresonant interaction, when the driven-pump carrier frequency is larger than all high eigenfrequencies of magnetized plasma, and resonant interaction, when the carrier frequency is equal to some of the higher magnetized plasma eigenfrequencies, are studied. Excitations of upper-hybrid, lower-hybrid, and oblique Langmuir waves coupled to ion-acoustic waves are considered. In all of these cases threshold values and parametric growth rates are obtained and compared with the case of a monochromatic driver pump (constant amplitude and phase). The relevance of the results obtained is discussed for laser and microwave-plasma interactions.

I. INTRODUCTION

The theory of parametric interaction of monochromatic driver pumps with plasmas is relatively well developed.[1] On the other hand, there exists a lot of experimental data which cannot be explained within the framework of a monochromatic driver-pump parametric theory, namely, those data directly connected with nonmonochromaticity (noncoherence) of electromagnetic radiation generators used in laser fusion,[2] magnetically confined plasmas heating,[3,4] or in ionospheric experiments.[5]

The study of parametric phenomena in plasmas produced by the action of nonmonochromatic driver pumps is primarily motivated by two reasons. The first is embodied in the fact that all real generators of electromagnetic radiation are inherently nonmonochromatic (noncoherent) with some frequency bandwidth $\Delta\omega_0$. If $\Delta\omega_0/\gamma > 1$ the frequency bandwidth of the driver pump plays a crucial role in the development of parametric processes in plasmas (γ is the parametric growth rate). The second reason is that all real generators operate with stochastically[6–8] or dynamically[9] modulated amplitude and frequency (or phase) which again results in nonmonochromaticity of the driver pump. Also, as will be shown in Part II (see following article)[10] simultaneous action of several monochromatic driver pumps on plasma can be reduced under certain circumstances to the case of a single nonmonochromatic driver pump. Besides these, in experimental situations the action of monochromatic electromagnetic radiation on plasmas is limited in time or occurs as pulse trains[10] of monochromatic radiation impinging on the plasmas. Both of these cases could be considered as special forms of amplitude modulation of the driver pumps which, consequently, appear to be quasimonochromatic.

The explanation of the influence of driver-pump finite bandwidth on parametric processes sometimes leads to obvious contradictions. For example, in Refs. 11 and 12, it is shown that in a laser-produced plasma the threshold for parametric instabilities of longitudinal excitations decreases with an increase in laser bandwidth, while in Refs. 5 and 13, the opposite is shown (the threshold value increases with an increase in laser bandwidth). Theoretical considerations based on stochastic variation of the driver-pump phase[2,7,8] have shown that in the case when the bandwidth $\Delta\omega_0$ is much larger than the parametric growth rate γ, the threshold of instabilities was significantly increased. For the stochastic amplitude modulation of a driver pump, an analogous result has been obtained in Ref. 6. Pustovalov *et al.*[9] have shown that the increase of the threshold value for parametric excitations with an increase in the generator bandwidth is not a general effect. For example, in the case of aperiodic instability, the finite bandwidth of the driver pump does not affect the threshold value in the case of amplitude modulation. However, if frequency modulation exists, the threshold is significantly increased.

From the above-listed results it can be seen that by variation of driver-pump coherence, we can control the development of parametric processes which play a crucial role in the laser-fusion problem or in laser or radio-frequency heating of magnetically confined plasmas. It is known, for example, that in laser fusion, scattering (Raman and Brillouin) parametric instabilities lead to the loss of laser radiation while absorptive (decay and oscillating two-stream) parametric instabilities could lead to the appearance of suprathermal particles[14] which, in turn, preheat the pellet core and reduce compression efficiency. In the second part of Ref. 10 it was shown that by convenient amplitude modulation of the driver pump, generation of suprathermal particles, due to the weak and strong parametric turbulence, could be reduced. In lower-hybrid[15] resonance heating of toroidal plasmas parametric instabilities could be responsible for nonaccessibility of the driver pump to the center. So, by changing the driver-pump coherence, undesirable parametric effects could be avoided or significantly reduced.

In this paper the study of longitudinal parametric exci-

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tations in magnetized plasma by the action of a pulse-operated (amplitude-modulated) driver pump is presented. The influence of phase modulation is also considered. In the theoretical treatment of this situation the main quantities are ulse duration (τ) of the driver pump, interpulse spacing (T), characteristic time (T_c) of the process in the plasma which is under consideration, and the carrier frequency (ω_0) of the driver pump. Further, the interpulse spacing (T) could be given in a stochastic or a dynamic manner. In the present paper only the dynamical case will be treated. In reference to the listed quantities it is to be noted that in the case of a long interpulse spacing (T) so that $T \gg T_R$ (T_R is the relaxation time of plasma processes) the most important phenomena take place during the pulse duration time (τ). Here, depending on the ratio of parametric growth rate (γ) and inverse pulse duration time ($1/\tau$), nonlinear effects are, or are not, to be taken into account.[16,17] If the parametric growth rate is much larger than inverse pulse duration time, the nonlinear effects occur during one pulse and the nonmonochromaticity of a driver pump does not play any significant role. The opposite case, when the saturation level is reached during the sequence of pulses, is more interesting with respect to the nonlinear parametric phenomena.[16,17]

In conclusion, let us note that the study of action of nonmonochromatic driver pumps on plasmas agrees with other studies considering the influence of real experimental situations on the development of parametric instabilities; for instance, the influence of inhomogeneity and boundaries,[1] the presence of electron beam and current,[18] finite wave-ength of a driver pump,[19] etc.

This paper is organized as follows. In Sec. II model equations based on the Vlasov–Poisson formalism are derived and compared with the case of a monochromatic driver pump (constant amplitude and phase). In Sec. III we deal with nonresonant interaction when the carrier frequency of a driver pump is assumed to be much larger than all the high eigenfrequencies of the magnetized plasma. In Sec. IV resonant interaction, when the carrier frequency of a driver pump is equal to the upper-hybrid, lower-hybrid, and oblique Langmuir wave frequencies, is studied. Discussion of results and principal conclusions are given in Sec. V.

II. MODEL EQUATIONS

Model equations for studying the parametric interaction of a nonmonochromatic driver pump in the form $\mathbf{E}(t) = \mathbf{E}_0(t)\sin[\omega_0 t + \phi(t)]$[$E_0(t)$ and $\phi(t)$ are the time-varying amplitude and phase of a driver pump and ω_0 its carrier frequency] with magnetized, homogeneous, electron–ion plasma are obtained utilizing a known plasma dielectric permittivity formalism[1] (see the Appendix). They have the following form:

$$n_e(\omega,\mathbf{k}) = R_e(\omega,\mathbf{k}) \times \sum_{l=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} n_i(\omega + n\Omega + l\omega_0,\mathbf{k}), \quad (1a)$$

$$n_i(\omega,\mathbf{k}) = R_i(\omega,\mathbf{k}) \times \sum_{l=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} n_e(\omega + n\Omega + l\omega_0,\mathbf{k}). \quad (1b)$$

Here $n_\alpha(\omega,\mathbf{k}) = e_\alpha \int d_\mathbf{p}\, \psi_\alpha(\omega,\mathbf{k},\mathbf{p})$ is charge density of α plasma component in a reference system oscillating with α particles and $\Psi_\alpha(\omega,\mathbf{k},\mathbf{p})$ is a space-time Fourier transform of a perturbed one-particle distribution function in an oscillating frame. Quantity $R_\alpha(\omega,\mathbf{k})$ is defined by $R_\alpha(\omega,\mathbf{k}) = \chi_\alpha(\omega,\mathbf{k}) \times [1 + \chi_\alpha(\omega,\mathbf{k})]^{-1}$, where $\chi_\alpha(\omega,\mathbf{k})$ is the linear susceptibility of the α plasma component [in our case it is an electron ($\alpha \equiv e$) and ion component ($\alpha \equiv i$)]. If the nonperturbed distribution function $f_\alpha^{(0)}$ is Maxwellian for longitudinal dielectric permittivity of magnetized plasma we have[20]

$$\epsilon(\omega,\mathbf{k}) = 1 + \chi_e(\omega,\mathbf{k}) + \chi_i(\omega,\mathbf{k}) = 1 + \sum_\alpha \frac{\omega_{p\alpha}}{kV_{T\alpha}}\Big(1 - \sum_{n=-\infty}^{+\infty} \frac{\omega}{\omega - n\Omega_\alpha} \times A_{|n|}(Z_\alpha) J_+ (\beta_n^{(\alpha)})\Big). \quad (2)$$

In (2), function J_+ is defined by

$$J_+(x) = x \exp\left(\frac{x^2}{2}\right) \int_{+i\infty}^{x} \exp\frac{t^2}{2}\, dt,$$

$$Z_\alpha = k_\perp^2 \rho_\alpha^2$$

$$\beta_n^{(\alpha)} = (\omega - n\Omega_\alpha)/|k_z| V_{T\alpha}$$

and

$$A_n(z) = I_n(z)\exp(-z).$$

In (z) is the modified Bessel's function and ρ_α and $V_{T\alpha}$ are the Larmor radius and thermal velocity of α particles, respectively. Here $k_\perp$ and k_z are the wavenumbers normal and parallel to magnetic field vector $\mathbf{B}_0$.

In the following sections (Sec. III and Sec. IV) as a concrete example of parametric excitations of magnetized-plasma high-frequency modes, we shall consider excitations of upper- and lower-hybrid waves and oblique Langmuir waves coupled to ion-acoustic waves. Oblique Langmuir waves cover all frequencies between electron-plasma and lower-hybrid waves depending on the angle of propagation θ. The corresponding dielectric permittivities of high-frequency modes could be easily obtained from (2) (see Ref. 20). They are, respectively,

$$\epsilon_{\rm UH}(\omega,\mathbf{k}) = 1 - \frac{\omega_{pe}^2}{\omega^2 - \Omega_e^2} + i\frac{\nu_{ei}}{\omega}\frac{\omega_{pe}^2(\omega^2 + \Omega_e^2)}{(\omega^2 - \Omega_e^2)^2}, \quad (3)$$

$$\epsilon_{\rm LH}(\omega,\mathbf{k}) = 1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2} + i\frac{\omega_{pe}^2 \nu_{ei}}{\omega^3}\frac{k_z^2}{k^2}, \quad \Omega_e \gtrsim \omega_{pe}, \quad (4)$$

$$\epsilon_{\rm OL}(\omega,\mathbf{k}) = 1 - \frac{\omega_{pe}^2 \cos^2\theta}{\omega^2} + i\frac{\omega_{pe}^2 \nu_{ei}}{\omega^3}\cos^2\theta, \quad \Omega_e \gtrsim \omega_{pe}. \quad (5)$$

Ion-acoustic wave dielectric permittivity has the form ($\omega \gg \Omega_i$, where Ω_i is the ion-cyclotron frequency)

$$\epsilon_{\rm IA}(\omega,\mathbf{k}) = 1 + \frac{1}{k^2 r_{De}^2} - \frac{\omega_{pi}^2}{\omega^2} + i\frac{8}{5}\frac{\omega_{pi}^2}{\omega^5}\nu_{ii} k_z^2 V_{Ti}^2. \quad (6)$$

In (3) and (5) ν_{ei} is the electron–ion collision frequency and in (6) ν_{ii} is the ion–ion collision frequency. For the sake of simplicity in the above dielectric permittivities only collisional dissipation is included. Noncollisional dissipation (Landau and cyclotron) can be found, for example, in Refs. 20 and 21.

In system (1) I_n denotes the coefficient of interaction given by

$$I_n^{(l)} = \frac{1}{T}\int_0^T dt\, J_l[\mu(t)]\exp[il\phi(t)]\exp(-in\Omega t). \quad (7)$$

In obtaining system (1) it was assumed that $E_0(t)$ and $\phi(t)$ are periodic functions with period $T = 2\pi/\Omega$ (Ω is modulation frequency). Quantity $\mu(t)$ is the coupling coefficient having the form

$$\mu(t) = [ekE_0(t)/m_e\omega_0^2]f^{1/2}(\theta,t), \quad (8)$$

where k is the wavenumber of parametrically excited waves and e and m_e are the charge and mass of an electron, respectively. The function $f(\theta,t)$ describes the geometry of parametric interaction. If the driver-pump electric field vector has arbitrary position in space $\mathbf{E}_0(t) = \{E_{0x}(t),E_{0y}(t),E_{0z}(t)\}$, the magnetic field vector is given by $\mathbf{B}_0 = \{0,0,B_0\}$, and the wave vector of excited waves is given by $\mathbf{k} = \{0,k_y,k_z\}$, then the function $f(\theta,t)$ obtains the following form:

$$f(\theta,t) = \left(\frac{E_{0z}(t)}{E_0(t)}\cos\theta + \frac{\omega_0^2}{\omega_0^2 - \Omega_e^2}\frac{E_{0y}(t)}{E_0(t)}\sin\theta\right)^2 + \left(\frac{E_{0x}(t)}{E_0(t)}\frac{\omega_0\Omega_e}{\omega_0^2 - \Omega_e^2}\sin\theta\right)^2. \quad (9)$$

In (9) θ is the angle between vectors $\mathbf{k}$ and $\mathbf{B}_0$ and Ω_e is the electron-cyclotron frequency. In what follows we shall be interested in the interaction of extraordinary $X(E_0 = \{0,E_0,0\})$ and ordinary $O(E_0 = \{0,0,E_0\})$ driver pumps at normal incidence $k_0 = \{k_0,0,0\}$ with magnetized plasma. ($\mathbf{k}_0$ is the wave vector of the driver pump. Here, however, the driver pump is treated in dipole approximation $|k_0| \sim 0$.) The corresponding coupling coefficients are

$$\mu(0) = k(r_{De}^2 + r_{Di}^2)^{1/2}(\omega_{pe}/\omega_0)^2\cos\theta \times \frac{E_0(t)}{[4\pi n_e(T_e + T_i)]^{1/2}}, \quad (10)$$

$$\mu(X) = k(r_{De}^2 + r_{Di}^2)^{1/2}(\omega_{pe}/\omega_0)^2\sin\theta \times \frac{\omega_0^2}{\omega_0^2 - \Omega_e^2}\frac{E_0(t)}{[4\pi n_e(T_e + T_i)]^{1/2}}. \quad (11)$$

Note that (8)–(11) are derived for the case $|\omega_0 - \Omega_e| \gg \Omega$. The influence of nonmonochromaticity of a driver pump on parametric processes at electron-cyclotron resonance was considered in Ref. 4.

As a concrete example of time dependence of $E_0(t)$ we shall study the amplitude modulation in the form of a rectangular pulse train with pulse duration τ and interpulse spacing T,

$$E_0(t) = \begin{cases} E_0 & nT < t < \tau + nT \\ 0 & \tau + nT < t < (n+1)T \end{cases} \quad n = 0,1,2\ldots. \quad (12)$$

For the time dependence of phase $\phi(t)$ we shall assume

$$\phi(t) = \phi_0, \quad (13)$$

$$\phi(t) = \alpha\sin\Omega t. \quad (14)$$

Now for the case of amplitude modulation given by (12) and constant phase (13) for the coefficients of nonlinear interaction we obtain the following forms (see Ref. 22):

$$I_n^{(0)} \equiv I_n(l=0) = -[1 - J_0(\mu_0)]\frac{\sin n\pi(\tau/T)}{n\pi} \times \exp[in\pi(\tau/T)], \quad n \neq 0, \quad (15)$$

$$I_0^{(0)} \equiv I_0(l=0) = 1 - (\tau/T)[1 - J_0(\mu_0)], \quad (16)$$

$$I_n^{(l)} \equiv I_n(l \neq 0) = J_l(\mu_0)\frac{\sin n\pi(\tau/T)}{n\pi} \times \exp\left(il\phi_0 - in\pi\frac{\tau}{T}\right). \quad (17)$$

If amplitude modulation is in the form of rectangular pulse train (12) and the phase modulation sinusoidal (14) coefficients of interaction are

$$I_n(l=0) = J_l(\mu_0)\delta_{n,0}, \quad (18)$$

$$I_n(l \neq 0) = J_l(\mu_0)J_n(l\alpha). \quad (19)$$

In (15)–(19) the time-independent coupling coefficient μ_0 is obtained from (8) and (9) substituting $E_0(t)$ with E_0. Here $\delta_{n,0}$ is the Kronecker symbol.

The system of the model equations (1) describes in a general manner the interaction of a nonmonochromatic driver pump with the electron and ion plasma components at frequencies ω and $\omega_1 = \omega + n\Omega + l\omega_0$. In the case of excitation of high- and low-frequency magnetized plasma modes, $\omega_1 = \omega_1(\mathbf{k})$ and $\omega = \omega(\mathbf{k})$ represent their linear dispersion relations, respectively. By system (1), as well, excitation of two high-frequency magnetized plasma modes is described, in which case both $\omega = \omega(\mathbf{k})$ and $\omega_1 = \omega_1(\mathbf{k})$ are linear dispersion relations of high-frequency magnetized plasma modes.

System (1) could be easily reduced to the case of a monochromatic driver pump[1] by putting $\mathbf{E}_0(t) = \mathbf{E}_0[\mathbf{E}_0$ is $\mathbf{E}_0(t)$ in $t = 0]$, $\phi(t) \equiv 0$, and $n = 0$. Then the coefficient of nonlinear interaction $I_0 \equiv J_n(\mu_0)$, where J_n is Bessel's function of order "n" and μ_0 is the coupling coefficient evaluated for $\mathbf{E}_0(t) = \mathbf{E}_0$. Further, if there is no driver pump ($J_n = 0$ for $n \neq 0$ and $J_0 = 1$, system (1) describes longitudinal eigenmodes of magnetized plasma.[20,21]

In the following, model equations (1) will be analyzed for the case of a transparent plasma when $\omega_0 \gg \omega_H$ (ω_H is the high eigenfrequency of magnetized plasma) and for the resonant case when $\omega_0 \sim 2\omega_H$ and $\omega_0 \sim \omega_H$.

A. Nonresonant case

Here we shall assume that the carrier frequency of a driver pump is much larger than all eigenfrequencies of a magnetized plasma. Now, the relevant responses of electron and ion plasma components are $n_{e,i}(\omega,\mathbf{k})$ and $n_{e,i}(\omega + n\Omega,\mathbf{k})$. Consequently in system (1) we are to put $l \equiv 0$ so that the following system describing the nonresonant interaction of a nonmonochromatic driver pump with magnetized plasma is obtained:

$$n_e(\omega,\mathbf{k}) = R_e(\omega,\mathbf{k})\sum_{n=-\infty}^{+\infty} I_n^{(0)} n_i(\omega + n\Omega,\mathbf{k}),$$
$$n_i(\omega,\mathbf{k}) = R_i(\omega,\mathbf{k})\sum_{n=-\infty}^{+\infty} I_n^{(0)} n_e(\omega + n\Omega,\mathbf{k}). \quad (20)$$

Here the $\Gamma_n^{(0)}$ are coefficients of nonresonant parametric interaction [see (7), (15), and (16)].

Let us note that in the nonresonant case [as is seen from (7)], modulation of a driver-pump phase does not play any role in the development of parametric processes. The system of model equations (20) is formally identical to Silin's model equations[1] for the case of resonant parametric interaction of a monochromatic driver pump with plasma if we substitute $n\Omega$ by $n\omega_0$ and put $\Gamma_n^{(0)} \equiv J_n(\mu_0)$. However, the system (20) predicts (as will be shown in Sec. III) a variety of parametric excitations in contrast to the monochromatic driver pump-plasma nonresonant interaction when plasma appears to be stable.[1]

B. Resonant case

First, we shall consider the case when $\omega_0 \sim 2\omega_H$ ($\omega \sim \omega_H \gg n\Omega$), i.e., summational parametric resonance in magnetized plasma[23] (in the case of isotropic plasma this is two-plasmon parametric excitation[1]). The relevant responses are $n_{i,e}(\omega + n\Omega,\mathbf{k})$ and $n_e(\omega + n\Omega - \omega_0,\mathbf{k})$. The corresponding model equations take the form:

$$n_e(\omega,\mathbf{k}) = R_e(\omega,\mathbf{k}) \sum_{n=-\infty}^{+\infty} \Gamma_n^{(0)} n_i(\omega + n\Omega,\mathbf{k}),$$

$$n_i(\omega,\mathbf{k}) = R_i(\omega,\mathbf{k}) \times \sum_{l=0,-1} \sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} n_e(\omega + n\Omega + l\omega_0,\mathbf{k}). \quad (21)$$

If $\omega_0 \sim \omega_H$ ($\omega \ll \omega_H, \omega + n\Omega \ll \omega_H$), relevant responses are $n_{i,e}(\omega + n\Omega,\mathbf{k})$ and $n_e(\omega + n\Omega + \omega_0,\mathbf{k})$. The corresponding model equations are

$$n_e(\omega,\mathbf{k}) = R_e(\omega,\mathbf{k}) \sum_{n=-\infty}^{+\infty} \Gamma_n^{(0)} n_i(\omega + n\Omega,\mathbf{k}),$$

$$n_i(\omega,\mathbf{k}) = R_i(\omega,\mathbf{k}) \times \sum_{l=0,\pm 1} \sum_{n=-\infty}^{\infty} \Gamma_n^{(l)} n_e(\omega + n\Omega + l\omega_0,\mathbf{k}). \quad (22)$$

The most general case of resonance is $p\omega_0 \sim \omega_H$. However, if $p \neq 1$ threshold values are significantly higher and we shall restrict ourselves to the case $p = 1$.

III. NONRESONANT INTERACTION

In nonresonant interaction we shall study excitation of two high-frequency modes: (a) ($n\Omega \sim 2\omega_H, \omega \sim \omega_H$), excitation of the high-frequency mode coupled to low-frequency modes; (b) ($n\Omega \sim \omega_H, \omega \ll \omega_H$); and finally (c) excitation of two low-frequency magnetized plasma modes ($n\Omega \sim 2\omega_L$).

(a) Eliminating $n_i(\omega + n\Omega,\mathbf{k})$ from (1b) and substituting it into (1a) as a condition that system (1) has a nontrivial solution, the following dispersion relation is obtained:

$$D^{(0)}D^{(-n)} = \sum_{m=-\infty}^{+\infty} |\Gamma_m^{(0)}|^2 |\Gamma_{m+n}^{(0)}|^2 \times \left(\frac{m_e}{m_i}\right)^2 \left(\frac{n^2}{(2m+n)^2}\right)^2. \quad (23)$$

Here

$$D^{(n)} = 1 + \chi_e(\omega + n\Omega,\mathbf{k}) - \sum_{p=-\infty}^{+\infty} |\Gamma_{n-p}^{(0)}|^2 R_i(\omega + p\Omega,\mathbf{k}). \quad (24)$$

Dispersion relation (23) describes a so-called strong coupling of plasma modes (the dispersion features of excited modes are modified by the presence of the driver pump). As is seen from (24) modification is realized through the ion-plasma component. A weak mode-coupling dispersion relation is found from (23) by putting $m = 0$:

$$\epsilon(\omega,\mathbf{k})\epsilon(\omega - n\Omega,\mathbf{k}) = |\Gamma_n^{(0)}|^2 (m_e/m_i)^2. \quad (25)$$

In the case of excitation of two oblique Langmuir waves with frequency $\omega = \omega_{pe}|\cos\theta|$ (excitation of two upper-hybrid waves was considered in Ref. 24) from (25) for parametric growth rate

$$\gamma = \frac{1}{8}\omega_{pe}|\cos\theta| \left(\frac{m_e}{m_i}\right) k^2 (r_{De}^2 + r_{Di}^2) \left(\frac{\omega_{pe}}{\omega_0}\right)^4 \times f(\theta) \left|\frac{\sin n\pi(\tau/T)}{n\pi}\right| \frac{E_0^2}{4\pi n_e (T_e + T_i)} \quad (26)$$

is obtained. In (26) we utilized expression (15) with $\mu_0 \ll 1$ and assumed that $\gamma \gg \gamma_H$, where γ_H is the linear damping rate of the oblique Langmuir wave. Minimum threshold for this excitation is reached if $n = 1$ and $T = 2\tau$:

$$\frac{E_{0,\mathrm{THR}}^2}{4\pi n_e (T_e + T_i)} = \frac{8\pi}{k^2 (r_{De}^2 + r_{Di}^2) f(\theta)} \times \frac{\gamma_H}{\omega_{pe}|\cos\theta|} \left(\frac{\omega_0}{\omega_{pe}}\right)^4 \left(\frac{m_i}{m_e}\right). \quad (27)$$

Note that in the case of an X driver pump $f(\theta) \equiv \cos\theta$ and in the case of an O driver pump $f(\theta) \equiv \sin\theta$.

(b) If $n\Omega \sim \omega_H$ ($\omega \ll \omega_H$) excitation of the high-frequency magnetized plasma mode coupled to a low-frequency mode is possible. In this case the dispersion relation obtained from system (1) [taking into account that dominant R quantities are $R_e(\omega + n\Omega,\mathbf{k})$ and $R_e(\omega,\mathbf{k})$] has the following form:

$$1 = R_i^{(0)} [\, |\Gamma_0^{(0)}|^2 R_e^{(0)} + |\Gamma_n^{(0)}|^2 (R_e^{(n)} + R_e^{(-n)}) \,],$$

$$R_\alpha^{(n)} = \chi_\alpha(\omega + n\omega_0,\mathbf{k}) [\, 1 + \chi_\alpha(\omega + n\omega_0,\mathbf{k}) \,]^{-1}. \quad (28)$$

Dispersion relation (28) describes strong ($\Gamma_0^{(0)} \sim 0$) and weak ($\Gamma_0^{(0)} \neq 0$) coupling of the excited waves. In the latter case it reduces to a dispersion relation for decay and oscillating two-stream instabilities in plasma upon the action of a nonmonochromatic driver pump. For decay instabilities the anti-Stokes' line $R_e^{(n)}$ could be neglected. Then, for the parametric growth rate of the excitation of an oblique Langmuir wave coupled to an ion-acoustic wave ($\gamma > \gamma_H, \gamma_S$),

$$\gamma = \frac{1}{8}(\omega_{pe}|\cos\theta|\omega_s)^{1/2} (k r_{De}) \left|\frac{\sin n\pi(\tau/T)}{n\pi}\right| \times \left(\frac{\omega_{pe}}{\omega_0}\right)^4 f(\theta) \frac{E_0^2}{4\pi n_e T_e} \quad (29)$$

is obtained. In the case of excitation of the upper-hybrid mode in (29), $(\omega_{pe}|\cos\theta|\omega_S)^{1/2}$ is to be substituted for by

$(\omega_{UH}\omega_S)^{1/2}(\omega_{pe}/\omega_{UH})$. Minimum threshold is reached for $n = 1$ and $T = 2\tau$:

$$\frac{E^2_{0,\mathrm{THR}}}{4\pi n_e T_e} = \frac{8\pi}{k r_{De} f(\theta)}\left(\frac{\gamma_H \gamma_S}{\omega_{pe}|\cos\theta|\omega_S}\right)^{1/2}\left(\frac{\omega_0}{\omega_{pe}}\right)^4. \quad (30)$$

In (30) the threshold value is a function of $(\gamma_H\gamma_S/\omega_H\omega_S)^{1/2}$ in contrast to the resonant case with a monochromatic driver pump, where it is a function of $[(\gamma_H/\omega_H)(\gamma_S/\omega_S)]$. Consequently parametric processes described by (26) and (29) (and processes similar to them) are weaker than the corresponding ones in the resonant case.

(c) If $n\Omega \sim 2\omega_L$ $(\omega \sim \omega_L)$ (ω_L is the low eigenfrequency of magnetized plasma) in system (1) the dominant R quantities are $R_{e,i}(\omega,\mathbf{k})$ and $R_{e,i}(\omega - n\Omega,\mathbf{k})$ so that the dispersion relation obtains the following form:

$$\epsilon(\omega,\mathbf{k})\epsilon(\omega - n\Omega,\mathbf{k}) = 4|I_n^{(0)}|^2\chi_e(\omega - n\Omega,\mathbf{k})\chi_e(\omega,\mathbf{k}) \times\chi_i(\omega - n\Omega,\mathbf{k})\chi_i(\omega,\mathbf{k}). \quad (31)$$

As a concrete example, here we shall study excitation of two low-frequency ion-acoustic waves $[\omega_S = k v_S \cos\theta, v_S = (T_e/m_i)^{1/2}]$. The parametric growth rate is

$$\gamma = \frac{1}{4}\omega_s|\cos\theta|\left|\frac{\sin n\pi(\tau/T)}{n\pi}\right|\left(\frac{\omega_{pe}}{\omega_0}\right)^4 \times f(\theta)\frac{E_0^2}{4\pi n_e T_e} - \gamma_S, \quad (32)$$

and the minimum threshold $(n = 1, T = 2\tau)$ is

$$\frac{E^2_{0,\mathrm{THR}}}{4\pi n_e T_e} = \frac{4\pi}{f(\theta)}\frac{\gamma_S}{\omega_S|\cos\theta|}\left(\frac{\omega_0}{\omega_{pe}}\right)^4. \quad (33)$$

In (32) γ_S is the linear damping rate of a low-frequency ion-acoustic wave. In the case of excitation of high-frequency ion-acoustic waves $(\omega = k v_S, \omega \gg \Omega_i)$ in (32) and (33) $\cos\theta \equiv 1$.

In conclusion, let us note that the above-studied instabilities are a direct consequence of the nonmonochromaticity of a driver pump. In the case of a monochromatic driver pump that is identical with $\tau = T$ coefficients of nonlinear interaction, $I_n^{(0)} \equiv 0$ and coupling of plasma modes does not exist.

IV. RESONANT INTERACTION

Let us now consider the case when the carrier frequency of a driver pump is equal to some of the high eigenfrequencies of magnetized plasmas $(\omega_0 \sim \omega_H)$. Taking into account that [see 1(a)]

$$n_e(\omega + n\Omega + l\omega_0,\mathbf{k}) = R_e(\omega + n\Omega + l\omega_0,\mathbf{k}) \times \sum_{p=-\infty}^{+\infty} I_{p-n}^{(-l)} n_i(\omega + p\Omega,\mathbf{k}), \quad (34)$$

from (1b) we obtain

$$\begin{aligned} n_i(\omega,\mathbf{k}) = R_i(\omega,\mathbf{k}) &\sum_{n=-\infty}^{+\infty}\sum_{p=-\infty}^{+\infty} n_i(\omega + p\Omega,\mathbf{k}) \\ &\times [I_n^{(0)} I_{p-n}^{(0)} R_e(\omega + n\Omega,\mathbf{k}) \\ &+ I_n^{(1)} I_{p-n}^{(-1)} R_e(\omega + n\Omega + \omega_0,\mathbf{k}) \\ &+ I_n^{(-1)} I_{p-n}^{(1)} R_e(\omega + n\Omega - \omega_0,\mathbf{k})]. \end{aligned} \quad (35)$$

In (35) harmonics of modulation frequency $n\Omega$ can be larger, equal to, or smaller than the low eigenfrequency of the magnetized plasmas. All these cases are described by the following dispersion relation [obtained from (35) as a demand for the existence of a nontrivial solution]:

$$D(\omega,\mathbf{k}) = \frac{C_n(\omega,\mathbf{k})}{D(\omega + n\Omega,\mathbf{k})} + \frac{C_{-n}(\omega,\mathbf{k})}{D(\omega - n\Omega,\mathbf{k})}. \quad (36)$$

Here

$$\begin{aligned} D(\omega,\mathbf{k}) = &\frac{\epsilon(\omega,\mathbf{k})}{[1 + \chi_e(\omega,\mathbf{k})]^2} \\ &- \sum_{S=-\infty}^{+\infty} [R_e(\omega + S\Omega + \omega_0,\mathbf{k}) I_S^{(-1)} I_{-S}^{(+1)} \\ &+ R_e(\omega + S\Omega - \omega_0,\mathbf{k}) I_S^{(-1)} I_{-S}^{(1)}], \end{aligned} \quad (37)$$

and

$$\begin{aligned} C_n(\omega,\mathbf{k}) = &\sum_{S=-\infty}^{+\infty} [R_e(\omega + S\Omega + \omega_0,\mathbf{k}) I_{S-n}^{(1)} I_{-S}^{(-1)} \\ &+ R_e(\omega + S\Omega - \omega_0,\mathbf{k}) I_{S-n}^{(-1)} I_{-S}^{(+1)}] \\ &\times \sum_{p=-\infty}^{+\infty} [R_e(\omega + p\Omega + \omega_0,\mathbf{k}) I_{n-p}^{(-1)} I_p^{(1)} \\ &+ R_e(\omega + p\Omega - \omega_0,\mathbf{k}) I_{n-p}^{(1)} I_p^{(-1)}]. \end{aligned} \quad (38)$$

If modulation frequency is large enough $\Omega > \omega_L (n\Omega \gg \omega_L)$ (the case $n\Omega \sim 2\omega_S$ was considered in Ref. 25) in dispersion relation (36), dominant R_e quantities are $R_e(\omega \pm \omega_0,\mathbf{k})$ so that in (37) and (38) $s \equiv 0$ and $p \equiv 0$. Then, (36) is reduced to

$$D(\omega,\mathbf{k}) = 0. \quad (39)$$

Neglecting in (39) $R_e(\omega + \omega_0,\mathbf{k})$ (the case of decay parametric instabilities) for the parametric growth rate of the excitation of an oblique Langmuir wave coupled to an ion-acoustic wave we obtain ($\gamma \gg \gamma_H, \gamma_S$, where γ_H and γ_S are linear damping rates of oblique Langmuir and ion-acoustic waves, respectively):

$$\gamma^2 = \frac{1}{4}\omega_{pe}|\cos\theta|\omega_S\left(\frac{\omega_S}{\omega_{pi}}\right)^2\frac{|I_0^{(1)} I_0^{(-1)}|}{k^4 r_{De}^4}. \quad (40)$$

In the case of an amplitude-modulated driver pump [see (12)], $|I_0^{(1)} I_0^{(-1)}| = (\mu_0^2/4)(\tau^2/T^2)$ and, in the case of sinusoidal phase modulation [see (14)], $|I_0^{(1)} I_0^{(-1)}| = (\mu_0^2/4) J_0^2(\alpha)$. Taking this into account, from (40) we obtain $(\mu_0 \ll 1)$:

$$\gamma = \frac{1}{4}(\omega_{pe}|\cos\theta|\omega_S)^{1/2}\left(\frac{\omega_S}{\omega_{pi}}\right)\frac{1}{|\cos\theta|^2}\frac{1}{k r_{De}} \times f^{1/2}(\theta)\begin{Bmatrix}\tau/T \\ J_0(\alpha)\end{Bmatrix}\frac{E_0}{(4\pi n_e T_e)^{1/2}}, \quad (41)$$

where τ/T and $J_0(\alpha)$ corresponds to the amplitude (rectangular pulse train) and the sinusoidal phase modulation of the driver pump, respectively. It is evident that result (41) [dependence of γ on τ/T and $J_0(\alpha)$] is general, i.e., does not depend on a concrete pair of parametrically excited waves. Similarly, for threshold values,

$$\frac{E^2_{0,\mathrm{THR}}}{4\pi n_e T_e} = \begin{Bmatrix}T^2/\tau^2 \\ J_0^{-2}(\alpha)\end{Bmatrix}\left(\frac{E^2_{0,\mathrm{THR}}}{4\pi n_e T_e}\right)_{\mathrm{MON}} \quad (42)$$

is obtained. Here $(E^2_{0,\mathrm{THR}}/4\pi n_e T_e)_{\mathrm{MON}}$ is the threshold value evaluated for a monochromatic driver pump. In (41) and (42) the case of a monochromatic pump can be obtained by putting $\tau/T\equiv 1$ and $\alpha\equiv 0$. In the case of excitation of an upper-hybrid wave $(\omega^2_{UH}=\omega_{pe}+\Omega^2_e)$ coupled to an ion-acoustic wave $[\omega_S=kv_s, v_S=(T_e/m_i)^{1/2}]$ in (41), $(\omega_{pe}|\cos\theta|\omega_S)^{1/2}(1/|\cos\theta|^2)$ is to be replaced by $(\omega_{UH}\omega_S)^{1/2}(\omega_{pe}/\omega_{UH})^3$ and in the case of excitation of a lower-hybrid wave $\{\omega_{LH}=\omega_{pi}/[1+(\omega^2_{pe}/\Omega^2_e)]^{1/2}\}$ coupled to an ion-acoustic wave by $(\omega_S\omega_{LH})^{1/2}(\omega_{pe}/\omega_{LH})^2$.

Resonant excitation of two high-frequency magnetized plasma modes is described by model equations (21). Here, however we shall not study this interaction in detail because the coupling of modes is realized through the ion-plasma component [in the case of an infinite-wavelength driver pump ($\mathbf{k}_0=0$) and a homogeneous plasma], and consequently threshold values are higher and parametric growth rates lower by approximately an order of magnitude compared to ordinary parametric excitations. In the case of a monochromatic driver pump in magnetized plasma this excitation was considered in Ref. 23 and for an isotropic plasma in Ref. 26.

V. DISCUSSION AND CONCLUSION

In this paper a theory of parametric interaction of a nonmonochromatic driver pump with magnetized plasma is developed. In the nonresonant case, when $\omega_0 \gg \omega_H$, it was shown that a variety of parametric instabilities exist. All of these instabilities are direct consequences of the nonmonochromaticity of the driver pump (in our case nonmonochromaticity is caused by the time variation of amplitude and phase) and do not exist if amplitude and phase are constant, i.e., if the driver pump is monochromatic. It is to be noted, however, that minimum threshold values (27), (30), and (33) are obtained if $n=1$, which, in turn, means that $\Omega\sim\omega_H$. Such a high modulation frequency is difficult to realize, which reduces the significance of this process in experiments. In this sense the most important case is excitation of two low-frequency magnetized plasma modes (when $\Omega\sim 2\omega_L$) with weak linear damping rates which lead to lower threshold values. Also, due to the nonresonant excitations described above, a part of the external electric field energy could be deposited in the regions of magnetized plasma (in ionospheric experiments or in heating of magnetized plasma), where, without modulation of a driver-pump (amplitude and phase), nonlinear (as well as linear) absorption is impossible. On the other hand, by the proper choice of the modulation frequency ($\Omega=2\pi/T$) these effects could be avoided.

In this paper nonmonochromaticity of a driver pump is caused by time variation of its amplitude and phase. However, the main result of the paper could be generalized to include a finite-bandwidth driver pump. Namely, by pulse operation of a driver pump in a frequency domain we have n driver pumps with descrete frequencies $\omega_n=\omega_0\pm n\Delta\omega$ where $\Delta\omega\sim 1/T$. So, due to the pulse operation, the driver pump appears to be a multiple-line driver (for example, the HF laser). Taking into account that intensities of the lines decrease with n as $\sin n\pi(\tau/T)/n\pi$, the case of a pulse-operated driver pump could be reduced to the case of a finite-bandwidth driver pump with bandwidth $\Delta\omega_0\sim 1/\tau$. Setting the width of the nonlinear interaction γ (parametric growth rate) equal to $\Delta\omega$, we obtain

$$\tau/T\sim\gamma/\Delta\omega_0. \tag{43}$$

Expression (43) is valid only in the case $\gamma>\gamma_H,\gamma_L$. If $\gamma<\max(\gamma_H,\gamma_L)$ in (43) we have to use $\min(\gamma_H,\gamma_L)$ instead of γ (see Ref. 6). Putting (43) into (42) we obtain the general result of enhancement of the threshold for decay instabilities if $\Delta\omega_0>\gamma$. This result is also found in the case of random phase and amplitude modulation.[7,27]

In resonant interaction through decay parametric instabilities (as well as oscillating two-stream instabilities) the driver pump energy is deposited in particles leading to the appearance of hot electrons. In laser fusion it represents a serious problem in pellet design. Raising the threshold values for these processes [as can be seen from (42)] in laser–plasma interaction (in the upper-hybrid layer due to the self-induced magnetic field) the energy of hot electrons could be significantly reduced. The similar effect of a finite-bandwidth driver pump on production of hot electrons via linear conversion is observed in Refs. 27 and 28.

Recently[29] a very interesting experiment on enhancement of ion heating due to the finite bandwidth of a driver pump at lower-hybrid resonance has been reported. In this experiment $\gamma/\Delta\omega_0\sim 1$ so that threshold remained the same as compared to the case of a monochromatic driver pump. In order to explain such experiments a development of nonlinear theory is needed. Only in the framework of nonlinear theory it is possible to find dependence of saturation levels of excited waves (as well as distribution of the energy over particles) on the bandwidth, or more generally, on the nonmonochromaticity of a driver pump.

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APPENDIX: DERIVATION OF MODEL EQUATIONS

A homogeneous, fully ionized plasma in interaction with a nonmonochromatic (amplitude- and/or phase-modulated) driver pump $\mathbf{E}(t)=\mathbf{E}_0(t)\sin[\omega_0 t+\phi(t)]$ is considered. The model equations describing this interaction can be derived from the Vlasov kinetic equations in the following form:

$$\frac{\partial f_\alpha}{\partial t}+\nabla_r f_\alpha+\frac{e_\alpha}{c}(\mathbf{V}\times\mathbf{B}_0)\nabla_p f_\alpha + e_\alpha\mathbf{E}(\mathbf{r},t)\nabla_p f_\alpha=0,\quad \alpha=e,i. \tag{A1}$$

Here $f_\alpha\equiv f_\alpha(\mathbf{r},\mathbf{p},t)$ is a one-particle distribution function of an "α" plasma component, $\mathbf{V}_\alpha$ is a velocity of α particles, and $\mathbf{p}_\alpha$ is the corresponding impulse (linear momentum). ∇_r and ∇_p are gradient operators in real and impulse space. The electric field in plasma $\mathbf{E}(\mathbf{r},t)$ has two components—external $\mathbf{E}(t)$ and self-consistent $\mathbf{E}^{(1)}(\mathbf{r},t)$ (which could be obtained from the Poisson equation). The perturbed plasma state described by the $f_\alpha^{(1)}$ distribution function is governed by the following equation (after putting the explicit form of the self-consistent field $\mathbf{E}^{(1)}$):

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + \mathbf{V}_\alpha \nabla_r f_\alpha^{(1)} - \nabla_p f_\alpha^{(0)} \nabla_{r\alpha} \sum_\beta \int d\mathbf{r}_\beta d\mathbf{p}_\beta \frac{e_\alpha e_\beta}{|\mathbf{r}_\alpha - \mathbf{r}_\beta|} f_\beta^{(1)}$$
$$= -e_\alpha [\mathbf{E}(t) + (1/c)(\mathbf{V}_\alpha \times \mathbf{B}_0)] \nabla_{p\alpha} f_\alpha^{(1)}. \quad \text{(A2)}$$

In (A2) the summation is over all plasma components.

Writing Eq. (A2) in the frame of reference oscillating with α particles, one obtains (after applying the Fourier transform in $\mathbf{r}$ space)

$$\frac{\partial \Psi_\alpha}{\partial t} + i\mathbf{k}\mathbf{V}_\alpha \Psi_\alpha + \frac{e_\alpha}{c}(\mathbf{V}\times\mathbf{B}_0)\nabla_p \Psi_\alpha$$
$$= i\mathbf{k}\nabla_p f_\alpha^{(0)} \sum_\beta \frac{4\pi e_\alpha e_\beta}{k^2} \int d\mathbf{p}_\beta \Psi_\beta$$
$$\times \sum_{l=-\infty}^{+\infty} J_l[\mu(t)]\exp\{il[\omega_0 t + \phi(t)]\}, \quad \text{(A3)}$$

where $\Psi_\alpha \equiv \psi_\alpha(t,\mathbf{k},\mathbf{p})$ is the perturbed distribution function $f_\alpha^{(1)}(\mathbf{k},\mathbf{p},t)$ in the oscillating frame of reference. J_l is the Bessel's function of the order l and $\mu(t)$ represents a time-dependent coupling coefficient given by Eq. (8) (Sec. II).

Expanding $J_l \exp[il\phi(t)]$ in a Fourier series:

$$J_l[\mu(t)]\exp[il\phi(t)] = \sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} \exp(in\Omega t), \quad \text{(A4)}$$

$$\Gamma_n^{(l)} = \frac{1}{T}\int_0^T J_l[\mu(t)]\exp[il\phi(t)]\exp(-in\Omega t)\,dt. \quad \text{(A5)}$$

From (A3) we obtain

$$\frac{\partial \Psi_\alpha}{\partial t} + i\mathbf{k}\mathbf{v}\Psi_\alpha + \frac{e_\alpha}{c}(\mathbf{V}_\alpha \times \mathbf{B})\nabla_p \Psi_\alpha$$
$$- i\mathbf{k}\nabla_p f_\alpha^{(0)} \sum_\beta \frac{4\pi e_\alpha e_\beta}{k^2} \int d\mathbf{p}_\beta \Psi_\beta$$
$$\times \sum_{l=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} e^{il\omega_0 - in\Omega t} = 0. \quad \text{(A6)}$$

Further, applying a Fourier transform in time on (A6), and taking into account that $\alpha \equiv e,i$, the following system of equations is obtained:

$$-i\omega\Psi_e + i\mathbf{k}\mathbf{v}\Psi_e + \frac{e}{c}(\mathbf{V}\times\mathbf{B}_0)\nabla_p \Psi_e$$
$$= i\mathbf{k}\nabla_p f_e^{(0)} \left(\frac{4\pi e^2}{k^2}\int d\mathbf{p}\, \Psi_e + \frac{4\pi e_i e}{k^2}\int d\mathbf{p}\right.$$
$$\left.\times \sum_{l=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} I_n \Psi_i(\omega + n\Omega + l\omega_0, \mathbf{k})\right),$$

$$-i\omega\Psi_i + i\mathbf{k}\mathbf{v}\Psi_i + (e_i/c)(\mathbf{V}\times\mathbf{B}_0)\nabla_p \Psi_e^{(0)} \quad \text{(A7)}$$
$$= i\mathbf{k}\nabla_p f_i^{(0)} \left(\frac{4\pi e^2}{k^2}\int d\mathbf{p}\, \Psi_i + \frac{4\pi e_i e}{k^2}\int d\mathbf{p}\right.$$
$$\left.\times \sum_{l=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} \Psi_e(\omega + n\Omega + l\omega_0, \mathbf{k})\right).$$

Introducing the charge density of the α plasma component in an oscillating frame $n_\alpha(\omega,\mathbf{k}) = e_\alpha \int d\mathbf{p}_\alpha\, \Psi_\alpha(\mathbf{p},\omega,\mathbf{k})$, and repeating known procedure for obtaining longitudinal dielectric permittivity of magnetized plasma[20] one obtains (1) (Sec. II):

$$n_e(\omega,\mathbf{k}) + \chi_e(\omega,\mathbf{k})n_e(\omega,\mathbf{k})$$
$$= \chi_e(\omega,\mathbf{k}) \sum_{l=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} n_i(\omega + n\Omega + l\omega_0, \mathbf{k}), \quad \text{(A8)}$$

$$n_i(\omega,\mathbf{k}) + \chi_i(\omega,\mathbf{k})n_i(\omega,\mathbf{k})$$
$$= \chi_i(\omega,\mathbf{k}) \sum_{l=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} \Gamma_n^{(l)} n_e(\omega + n\Omega + l\omega_0, \mathbf{k}).$$

Here $\chi_\alpha(\omega,\mathbf{k})$ is the linear susceptibility of the α plasma component given by[20]

$$\chi_\alpha(\omega,\mathbf{k}) = \frac{4\pi e_\alpha^2}{k^2}\int d\mathbf{p} \sum_{n=-\infty}^{+\infty} J_n\left(k_\perp \frac{V_{\alpha\perp}}{\Omega_\alpha}\right)$$
$$\times \frac{k_z \nabla_{pz} f_\alpha^{(0)} + (n\Omega_\alpha / V_{\alpha\perp})\nabla_{p\perp} f_\alpha^{(0)}}{\omega - n\Omega_\alpha - k_z V_{\alpha z}}. \quad \text{(A9)}$$

For the case of a Maxwellian nonperturbed distribution function $f_\alpha^{(0)}$, (A9) is reduced to (2) (Sec. II). Note that the interaction of a nonmonochromatic driver pump with an isotropic plasma is also described formally by model equations (A8). Now, however

$$\chi_\alpha(\omega,\mathbf{k}) = \frac{4\pi e_\alpha^2}{k^2}\int \frac{d\mathbf{p}\,\mathbf{k}\nabla_p f_\alpha^{(0)}}{\omega - \mathbf{k}\mathbf{V}}, \quad \text{(A10)}$$

$$\mu(t) = \frac{e\mathbf{k}\mathbf{E}_0(t)}{m_e \omega_0^2}. \quad \text{(A11)}$$

The case of a nonmonochromatic driver pump in interaction with an isotropic plasma was studied in Ref. 22.

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