Unified theory of parametric excitations in magnetized plasma produced by the action of nonmonochromatic driver pump. II. Multiple driver pump

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Using the Vlasov–Poisson formalism a theory of parametric interaction of n driver pumps with magnetized plasmas is developed. The interaction when the plasma appears to be transparent to all driver pumps is studied. It is shown that in this case a variety of parametric instabilities exist which are absent in a single driver pump-plasma interaction. The resonant interaction is also considered. Threshold values and parametric growth rates are obtained for the case of excitation of electron-Bernstein modes coupled to ion-Bernstein modes. The relevance of the results obtained is discussed for electromagnetic wave-ionospheric plasma interaction and laboratory plasma experiments.

I. INTRODUCTION

An interest in multiple driver-pump nonlinear interaction with plasma began nearly two decades ago.[1] It was primarily motivated by problems of heating of laboratory plasmas as well as by ionospheric experiments.[2–7] Besides this, from the point of view of basic plasma physics, this problem is also very interesting because it leads to enrichment of the processes that take place during the interaction and to the appearance of new instabilities.[3] In Refs. 1, 2, and 5 this problem was studied from the aspect of laser-driven fusion where excitation of plasma waves is realized by beating two laser beams. In Refs. 4 and 7 a full version of the double-resonance parametric theory of isotropic plasma with emphasis on ionospheric applications was given.

In this paper a general theory of multiple driver-pump parametric excitations in a magnetized plasma is developed. It represents the complementary part to Ref. 8 where the parametric theory of a modulated driver pump in interaction with a magnetized plasma is given. As will be shown in the present paper, interactions of a modulated driver pump and a multiple driver pump with plasma are very similar and, under certain circumstances, they can be reduced to each other.

The organization of the paper is as follows: in Sec. II the model equations (derived from the Vlasov–Poisson formalism) for the interaction of multiple driver pumps with magnetized plasma are given. In Sec. III dispersion relations describing nonresonant (when the frequencies of all driver pumps are larger than the highest eigenfrequency of the magnetized plasma) and resonant (when the frequencies of the driver pumps are approximately equal to some of the high eigenfrequencies of the magnetized plasma) interaction are obtained. In Secs. IV and V we deal with nonresonant and resonant interaction in the case of excitation of electron- and ion-Bernstein modes. The results obtained in Ref. 8 with excitations of upper-hybrid, lower-hybrid, and oblique-Langmuir waves coupled to ion-acoustic waves are also applicable here if some trivial substitutions are made. In Sec. V emphasis is placed on double parametric resonance. Finally, in Sec. VI discussion of results and their relevance for ionospheric and laboratory plasma experiments are given.

II. VLASOV–POISSON FORMALISM OF MULTIPLE DRIVER-PUMP PARAMETRIC EXCITATIONS IN MAGNETIZED PLASMA

A homogeneous, magnetized plasma in interaction with $(N+1)$ driver pumps in the form

$$\mathbf{E}(t) = \sum_{n=0}^{N} \mathbf{E}_n \sin(\omega_n t + \varphi_n) \tag{1}$$

is considered. Using a well-known Vlasov–Poisson formalism[9] infinite systems of algebraic equations (see the Appendix),

$$n_e^{(s)} = -R_e^{(s)} \sum_{\substack{m,l_n=-\infty \\ n=1\cdots N}}^{+\infty} J_{s-m}(\mu) \prod_{n=1}^{N} J_{-l_n}(\mu^{(n)}) e^{il_n\varphi_n} \times n_i\Big(\omega + m\omega_0 + \sum_{n=1}^{N} l_n\omega_n, \mathbf{k}\Big), \tag{2a}$$

$$n_i^{(s)} = -R_i^{(s)} \sum_{\substack{m,l_n=-\infty \\ n=1\cdots N}}^{+\infty} J_{m-s}(\mu) \prod_{n=1}^{N} J_{l_n}(\mu^{(n)}) e^{il_n\varphi_n} \times n_e\Big(\omega + m\omega_0 + \sum_{n=1}^{N} l_n\omega_n, \mathbf{k}\Big), \tag{2b}$$

are obtained. Here $n_\alpha^{(s)} = n_\alpha(\omega + s\omega_0, \mathbf{k})$ and $n_\alpha(\omega,\mathbf{k}) = e_\alpha \int d\mathbf{p}\, \Psi_\alpha(\omega,\mathbf{k},\mathbf{p})$ is the time-space Fourier transform of charge density of α particles in their oscillating frame of reference. $\Psi_\alpha(\omega,\mathbf{k},\mathbf{p}_\alpha)$ is the corresponding one-particle distribution function in an oscillating frame. In (2) the phase of a main driver pump φ_0 is assumed to be zero as a reference phase (phases of "escort" pumps are denoted by φ_n, $n = 1 \cdots N$). Quantity $R_\alpha^{(s)}(\omega,\mathbf{k}) = R_\alpha(\omega + s\omega_0,\mathbf{k})$ is defined by

$$R_\alpha(\omega,\mathbf{k}) = \chi_\alpha(\omega,\mathbf{k})/[1 + \chi_\alpha(\omega,\mathbf{k})], \tag{3}$$

where $\chi_\alpha(\omega,\mathbf{k})$ is the linear susceptibility of the α plasma component given by[10]

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$$\chi_\alpha(\omega,\mathbf{k}) = \frac{4\pi e_\alpha^2}{k^2}\int d\mathbf{p} \sum_{n=-\infty}^{+\infty} J_n^2\left(k_\perp \frac{V_{\alpha\perp}}{\Omega_\alpha}\right) \times \frac{k_z \nabla_{p_z} f_\alpha^{(0)} + \dfrac{n\Omega_\alpha}{V_{\alpha\perp}} \nabla_{p_\perp} f_\alpha^{(0)}}{\omega - n\Omega_\alpha - k_z V_{\alpha z}}. \tag{4}$$

In (4) ω and $\mathbf{k}$ are the frequency and the wave vector of the excited magnetized plasma modes, respectively; and $k_\perp$ and k_z are the normal (to the magnetic field which is oriented along the z axis) and parallel wavenumbers and $V_{\alpha_\perp}$ and V_{α_z} are the normal and parallel components of velocity $\mathbf{V}_\alpha$, respectively. J_n is Bessel's function of the order "n" and e_α, Ω_α are the charge of the particle α and the electron-cyclotron frequency, respectively.

The system (2a) and (2b) describes coupling of electron and ion plasma components by an external multiple driver pump at $(\omega + s\omega_0, \mathbf{k})$ and $\left(\omega + m\omega_0 + \sum_{N}^{n=1} l_n\omega_n, \mathbf{k}\right)$.

The coefficient of nonlinear interaction is given by

$$I = \prod_{n=0}^{N} J_{-l_n}(\mu^{(n)})\exp(il_n\varphi_n), \tag{5}$$

where $\mu^{(n)}$ is the coupling coefficient:

$$\mu^{(n)} = (ekE_n/m_e\omega_n^2)[f^{(n)}(\theta,\varphi,\chi)]^{1/2}. \tag{6}$$

In (6) $f^{(n)}(\theta,\varphi,\chi)$ is a function describing the interaction geometry of the nth escort pump with plasma. The coupling coefficient for the main driver pump is $\mu^{(0)} \equiv \mu$. The function $f^{(n)}(\theta,\varphi,\chi)$ is given by (see Ref. 9)

$$\begin{aligned} f^{(n)}(\theta,\varphi,\chi) = {} & \{\cos\theta\cos\chi^{(n)} + [\omega_n^2/(\omega_n^2 - \Omega_e^2)] \\ & \times \sin\theta\sin\chi^{(n)}\cos\varphi^{(n)}\}^2 \\ & + \{\sin\theta\sin\chi^{(n)}\sin\varphi^{(n)} \\ & \times [\omega_n\Omega_e/(\omega_n^2 - \Omega_e^2)]\}^2. \end{aligned} \tag{7}$$

In (7) θ is the angle between $\mathbf{k}$ and $\mathbf{B}_0$, $\chi^{(n)}$ is the angle between $\mathbf{B}_0$ and $\mathbf{E}_n$, and $\varphi^{(n)}$ is the angle between the planes formed by vectors $\mathbf{k}$, $\mathbf{B}_0$ and $\mathbf{E}_n$, $\mathbf{B}_0$. For the geometry of interaction considered in Ref. 8, the expression (7) is reduced to one similar to (9) (Ref. 8). If the nondisturbed distribution function $f_\alpha^{(0)}$ is assumed to be Maxwellian, expression (4) reduces to (2) and (A9) (Ref. 8).

The system (2a) and (2b) with (4), (6), and (7) represents the most general treatment of parametric interaction of the main driver pump and N escort driver pumps with magnetized plasma. It could be very easily reduced to the known[9] case of interaction of a single driver pump with magnetized plasma (see Appendix).

As a concrete example of parametric excitations in multiple driver pump-magnetized plasma interaction we shall study excitations of electron- and ion-Bernstein modes as pure kinetic modes of magnetized plasma. Their corresponding dielectric permittivities could be obtained from (4) (see Ginzburg and Rukhadze[10]) in the form:

$$\epsilon_{\mathrm{EB}}(\omega,\mathbf{k}) = 1 - \frac{\omega_{pe}^2}{\omega}\sum_{n=-\infty}^{+\infty} \frac{n^2A_n(k^2\rho_e^2)}{k^2\rho_e^2(\omega - n\Omega_e)} + in!2^n \frac{k^2r_{De}^2}{(k^2\rho_e^2)^n}\frac{\nu_{ei}}{n|\Omega_e|}, \tag{8}$$

$$\epsilon_{\mathrm{IB}}(\omega,\mathbf{k}) = 1 + \frac{1}{k^2r_{De}^2} - \frac{\omega_{pi}^2}{\omega}\sum_{n=-\infty}^{+\infty}\frac{n^2A_n(k^2\rho_i^2)}{k^2\rho_i^2(\omega - n\Omega_i)} + i\frac{56}{5}\frac{\Omega_i\nu_{ii}}{\omega_{p_i}^2}k^2\rho_i^2. \tag{9}$$

Here $A_n(x) = I_n(x)\exp(-x)$, where $I_n(x)$ is the modified Bessel's function and ν_{ei} and ν_{ii} are the electron–ion and ion–ion collision frequencies, respectively. The modes (8) and (9) and those studied in Ref. 8 cover a very wide range of eigenfrequencies of magnetized plasma. All results obtained in Ref. 8 with modulated driver pump can be transferred to the case of a multiple driver pump studied in this paper by doing trivial substitutions.

In the case of two escort driver pumps

$$\mathbf{E}(t) = \mathbf{E}_0\sin\omega_0 t + \mathbf{E}_1\sin(\omega_1 t) + \mathbf{E}_2\sin\omega_2 t,$$

if

$$\omega_0 - \omega_1 = -(\omega_0 - \omega_2) = \Omega\ (\Omega \ll \omega_0)$$

and

$$\mathbf{E}_1 = \mathbf{E}_2 = \alpha\mathbf{E}_0,$$

we have

$$\mathbf{E}(t) = \mathbf{E}_0(1 + 2\alpha\cos\Omega t)\sin\omega_0 t.$$

This means that the case of two escort pumps could be reduced to the case of an amplitude-modulated driver pump (see Ref. 8).

In the following, however, we shall be interested in the interaction of two driver pumps with magnetized plasma. In this case interaction coefficients (5) are reduced to

$$I = J_{-l_0}(\mu)J_{-l_1}(\mu^{(1)}), \quad \varphi_0 = \varphi_1 \equiv 0. \tag{10}$$

Now the electric field of a driver pump is

$$\mathbf{E}(t) = \mathbf{E}_0\sin\omega_0 t + \mathbf{E}_1\sin\omega_1 t.$$

If

$$\omega_0 - \omega_1 = \Omega,\ (\Omega \ll \omega_0),\ \mathbf{E}_1 \sim \mathbf{E}_0,$$

then

$$\mathbf{E}(t) = 2\mathbf{E}_0\cos(\Omega t/2)\sin\omega_1 t$$

so that it could be, again, considered as a special case of amplitude modulation. The influence of the driver-pump phases in the case of two escort pumps was considered in Ref. 11.

III. DERIVATION OF NONLINEAR DISPERSION RELATIONS

In this section we shall derive dispersion relations for the nonresonant and resonant cases based on the system of model equations (2a) and (2b). Let us first consider the case when the driver-pump frequencies are higher than all high eigenfrequencies of a magnetized plasma. This is the case of a completely transparent plasma which appears to be stable upon the action of a single driver pump.[9] This case corresponds to the nonresonant case of Ref. 8 where the carrier frequency of the driver pump was larger than the eigenfrequencies of magnetized plasma. Model equations for the nonresonant double driver pump–plasma interaction can be obtained from (2a) and (2b) in the following form:

$$n_e(\omega,\mathbf{k}) = -R_e(\omega,\mathbf{k}) \sum_{s=-\infty}^{+\infty} I_{-s} n_i(\omega + s\Omega,\mathbf{k}),$$

$$n_i(\omega,\mathbf{k}) = -R_i(\omega,\mathbf{k}) \sum_{s=-\infty}^{+\infty} I_s n_e(\omega + s\Omega,\mathbf{k}), \tag{11}$$

$$I_s = J_{-s}(\mu^{(0)}) J_s(\mu^{(1)}).$$

System (11) describes the interaction of the double driver pump with magnetized plasma where $\Omega = \omega_0 - \omega_1$ and $\omega_0, \omega_1 \gg \omega_H$. If $\Omega \sim 2\omega_H$ and $\omega \sim \omega_H$, excitation of two high-frequency magnetized plasma modes is possible. By putting in (11) $s = 0, -1$ the following dispersion relation

$$\left(1 - R_e^{(0)} \sum_{s=-\infty}^{+\infty} I_s R_i^{(s)}\right)\left(1 - R_e^{(-1)} \sum_{s=-\infty}^{+\infty} I_{s+1} R_i^{(s)}\right)$$
$$= R_e^{(0)} R_e^{(-1)} \left(\sum_{s=-\infty}^{+\infty} I_s I_{s+1} R_i^{(s)}\right)^2 \tag{12}$$

is obtained. In Ref. 8 we did not treat the case of excitation of two high-frequency modes because of the difficulties in realizing high modulation frequency in experiments. Here, however, the modulation frequency Ω is formally identical to the beat frequency $\Omega = \omega_0 - \omega_1$ which could be easily changed in experiments.

If $\Omega \sim \omega_H$ and $\omega \ll \omega_H$ excitation of a high-frequency magnetized plasma mode coupled to some of the low-frequency modes is possible. Now, in system (11) relevant responses are $n_e(\omega \pm \Omega,\mathbf{k})$ and $n_{e,i}(\omega,\mathbf{k})$. The corresponding dispersion relation has the form:

$$1 = R_i(\omega,\mathbf{k})\{I_0^2 R_e^{(0)} + I_1^2 R_e^{(1)} + I_{-1}^2 R_e^{(-1)}\}. \tag{13}$$

In nonresonant double driver pump-plasma interaction excitation of two low-frequency magnetized plasma modes (ω_L) is possible if $\Omega \sim 2\omega_L$ and $\omega \sim \omega_L$. In this case dominant R quantities are $R_{e,i}^{(0)}$ and $R_{e,i}^{(-1)}$. From (11) the following dispersion relation is obtained:

$$\epsilon(\omega,\mathbf{k})\epsilon(\omega - \Omega,\mathbf{k}) = 4A_1 A_{-1} \chi_i(\omega,\mathbf{k})\chi_e(\omega - \Omega,\mathbf{k})$$
$$\times \chi_i(\omega,\mathbf{k})\chi_i(\omega - \Omega,\mathbf{k}). \tag{14}$$

In resonant interaction all driver-pump frequencies are close to some of the high eigenfrequencies of the magnetized plasma. In the case of double resonance (see Ref. 4) the system (2a) and (2b) is reduced to

$$n_e(\omega,\mathbf{k}) = -R_e(\omega,\mathbf{k}) \sum_{m,l_1=-\infty}^{+\infty} J_{-m}(\mu^{(0)}) J_{-l_1}(\mu^{(1)})$$
$$\times n_i[\omega + (m + l_1)\omega_0 + l_1\Omega,\mathbf{k}],$$
$$n_i(\omega,\mathbf{k}) = -R_i(\omega,\mathbf{k}) \sum_{m,l_1=-\infty}^{+\infty} J_m(\mu^{(0)}) J_{l_1}(\mu^{(1)})$$
$$\times n_e[\omega + (m + l_1)\omega_0 + l_1\Omega,\mathbf{k}]. \tag{15}$$

In (15) it is assumed that $\omega_1 = \omega_0 + \Omega$ and $\omega \sim \omega_L$, $\omega_0 \sim \omega_H$. Taking into account that relevant responses in (15) are $n_{i,e}(\omega,\mathbf{k})$ and $n_e(\omega \pm \omega_0,\mathbf{k})$ it turns out that $m + l_1 = 0, \pm 1$ and $l_1 = -2$(for $\Omega = \omega_L$) and $l_1 = -1$ for $\Omega = 2\omega_L$. The coupling of waves under $l_1 = -2$ is weaker and in what follows we shall assume $l_1 = -1$ and consequently $\Omega = 2\omega_L$ (see Ref. 4). Under these circumstances from (15) the following dispersion relation can be obtained:

$$D(\omega - \Omega,\mathbf{k}) D(\omega,\mathbf{k}) = \Upsilon_+(\omega - \Omega,\mathbf{k})\Upsilon_-(\omega,\mathbf{k}), \tag{16}$$

where

$$D(\omega,\mathbf{k}) = \frac{\epsilon(\omega,\mathbf{k})}{1 + \chi_e(\omega,\mathbf{k})} + \frac{1}{4}\chi_i(\omega,\mathbf{k}) \sum_{n=1}^{2} (\mu^{(n)})^2$$
$$\times \left(\frac{1}{\epsilon^{(+n)}} + \frac{1}{\epsilon^{(-n)}}\right), \tag{17}$$

$$\Upsilon_{\pm}(\omega,\mathbf{k}) = -\frac{1}{4}\chi_i(\omega,\mathbf{k})\mu^{(0)}\mu^{(1)}\left(\frac{1}{\epsilon^{(\pm 1)}} + \frac{1}{\epsilon^{(\mp 1)}}\right),$$
$$\epsilon^{(\pm n)} = \epsilon(\omega \pm \omega_n,\mathbf{k}), \quad \omega_n \equiv \omega_0, \omega_1. \tag{18}$$

The dispersion relation (16) is first obtained in Ref. 4 for the case of isotropic plasma. Here it is generalized to include constant magnetic fields. In (16) it is assumed that the driver pumps are relatively weak $\mu^{(0)}, \mu^{(1)} \ll 1$. The case when one driver pump is strong ($E_0^2/4\pi n_e T_e > 1$) and the other one is weak ($E_1^2/4\pi n_e T_e \ll 1$) is studied in Ref. 12.

IV. NONRESONANT EXCITATIONS

Neglecting the influence of the double driver pump on dispersion features of plasma, from dispersion relation (12),

$$\epsilon(\omega,\mathbf{k})\epsilon(\omega - \Omega,\mathbf{k}) = \left(\sum_{s=-\infty}^{+\infty} I_{s+1} I_s R_i^{(s)}\right)^2 \tag{19}$$

is obtained. In (19) it could be seen that coupling of two high-frequency magnetized plasma modes is realized through an ion plasma component and consequently relatively high thresholds and low growth rates are to be expected. Substituting for ω by $\omega + i\gamma$ and expanding plasma dielectric permittivity in Taylor series from (19) for the parametric growth rate ($\gamma \gg \gamma_H$, where γ_H is the linear damping rate of the high-frequency mode),

$$\gamma^2 = \frac{(\Sigma_{s=-\infty}^{+\infty} I_{s+1} I_s R_1^{(s)})^2}{(\partial R_e \epsilon_H/\partial\omega)_{\omega=\omega_H^{(1)}} (\partial R_e \epsilon_H/\partial\omega)_{\omega=\omega_H^{(2)}}} \tag{20}$$

is obtained. The corresponding threshold value could be obtained by putting $\gamma \to \gamma_H$ in (20). In the case of excitation of two electron-Bernstein modes [see (8)] with frequencies $\omega_H^{(1)} \sim n\Omega_e$ and $\omega_H^{(2)} \sim m\Omega_e$, (20) becomes $[R_1^{(s)} \equiv \chi_i(\omega,\mathbf{k})]$

$$\gamma = \Omega_e \left(\frac{\omega_{pi}}{\omega_{pe}}\right)^2 \left(\frac{mn}{n!m!}\right)^{1/2} \frac{1}{2^{(n+m)/2}} \frac{(k^2\rho_e^2)^{(n+m+2)/2}}{(kr_{De})^4}$$
$$\times \frac{A_n(k^2\rho_i^2)}{k^2\rho_i^2} |I_1 I_0|. \tag{21}$$

In the case of excitation of a high-frequency ($\Omega \sim \omega_H$) mode coupled to a low-frequency mode ($\omega \sim \omega_L$) from (13) for weak driver pumps ($\mu^{(0)}, \mu^{(1)} \ll 1$) we obtain

$$\frac{\epsilon(\omega,\mathbf{k})}{[1 + \chi_e(\omega,\mathbf{k})]^2} = -\left(\frac{\mu^{(0)}\mu^{(1)}}{4}\right)^2$$
$$\times \left(\frac{1}{\epsilon(\omega + \Omega,\mathbf{k})} + \frac{1}{\epsilon(\omega - \Omega,\mathbf{k})}\right). \tag{22}$$

Neglecting the anti-Stokes' line (under the assumption that $\gamma_H/\omega_L \ll 1$), we obtain, from (22):

$$\gamma^2 = \left(\frac{\mu^{(0)}\mu^{(1)}}{4}\right)^2 \times \frac{[1+\chi_e(\omega,\mathbf{k})]^2}{(\partial R_e \epsilon_H/\partial\omega)_{\omega=\omega_H}(\partial R_e \epsilon_L/\partial\omega)_{\omega=\omega_L}}. \quad (23)$$

If an electron-Bernstein mode ($\omega_H = n\Omega_e$) coupled to an ion-Bernstein mode ($\omega_L = m\Omega_i$) is excited, from (23),

$$\gamma = \frac{\mu^{(0)}\mu^{(1)}}{2k^2 r_{De}^2}\left(\frac{mn}{m!n!}\right)^{1/2} \frac{(\Omega_i\Omega_e)^{3/2}}{(\omega_{pi}\omega_{pe})} \frac{(k^2\rho_i^2)^{(m+1)/2}}{2^{m+n/2}} \times \frac{(k^2\rho_e^2)^{(n+1)/2}}{(kr_{De})^2(kr_{Di})^2} \quad (24)$$

is obtained.

If two low-frequency modes are excited, from (14) ($\gamma \gg \gamma_L, \gamma_L$ low-frequency-mode lilnear damping rate),

$$\gamma^2 = \frac{4I_1 I_{-1}\chi_e(\omega,\mathbf{k})\chi_e(\omega-\Omega,\mathbf{k})\chi_i(\omega,\mathbf{k})\chi_i(\omega-\Omega,\mathbf{k})}{(\partial R_e\epsilon_L/\partial\omega)_{\omega=\omega_L^{(1)}}(\partial R_e\epsilon_L/\partial\omega)_{\omega=\omega_L^{(2)}}} \quad (25)$$

is obtained. Expression (25), in the case of excitation of two ion-Bernstein modes with $\omega_L^{(1)} \sim n|\Omega_i|$ and $\omega_L^{(2)} \sim m|\Omega_i|$, reduces to:

$$\gamma = 2(I_1 I_{-1})^{1/2}\frac{1}{(kr_{De})^2}\left(\frac{\omega_{pi}}{\Omega_i}\right)^2\left(\frac{r_{Di}}{\rho_i}\right)^2 \times \frac{(mn)^{1/2}}{(m!n!)^{1/2}2^{(m+n)/2}}\frac{\Omega_i^3}{\omega_{pi}^2}\frac{(k^2\rho_i^2)^{(n+m+2)/2}}{(kr_{Di})^4}. \quad (26)$$

From (21), (24), and (26) it is apparent that for a single driver pump, parametric growth rates vanish and, consequently, coupling of the waves does not exist.

V. RESONANT EXCITATIONS

The threshold value problem for the double parametric resonance in isotropic plasma was treated in detail in Refs. 4 and 6. Dispersion relation (16) is, however, rather complex for analytical treatment of parametric growth rate and requires numerical study (see Ref. 20). Here, we shall give an estimate of parametric growth rate with a double resonance condition using some assumptions recently verified by experiments[13,14] of Wong *et al.* with ionospheric plasma. In Refs. 13 and 14 it was shown that under the condition of double resonance when $\omega_0 - \omega_1 \sim 2\omega_s$ (ω_s is the ion-acoustic wave frequency) amplitudes of the side bands ($\omega_0 \pm \omega_s,\mathbf{k}$), ($\omega_1 + \omega_s,\mathbf{k}$) are significantly smaller than the amplitude of the outer side band with frequency $\omega_1 - \omega_s$. Physically, it means that in interaction decay parametric processes take place. Taking into account these results, dispersion relation (16) could be significantly simplified and solved by a successive perturbation method if $(E_n^2/4\pi n_e T_e)(\omega_L/\gamma_H) \ll 1$ ($E_n \equiv E_0$, E_1 are the electric field amplitudes of the driver pumps). In the first approximation relation (17) has the form $[(\mu^{(0)}\mu^{(1)})^2 \sim 0]$

$$D(\omega-\Omega,\mathbf{k})D(\omega,\mathbf{k}) = 0. \quad (27)$$

Standard treatment of this equation gives the following solution for the parametric growth rate:

$$(\gamma+\gamma_L)(\gamma+\gamma_H) = \tfrac{1}{4}[1+R_e\chi_e(\omega,\mathbf{k})]^2 \times \sum_{n=1}^{2}(\mu^{(n)})^2\left[\left(\frac{\partial R_e\epsilon_H}{\partial\omega}\right)_{\omega=\omega_H} \times \left(\frac{\partial R_e\epsilon_L}{\partial\omega}\right)_{\omega=\omega_L}\right]^{-1}. \quad (28)$$

In (28),

$$\gamma_{H,L} = \frac{I_m\epsilon_{H,L}(\omega,\mathbf{k})}{[\partial R_e\epsilon_{H,L}(\omega,\mathbf{k})/\partial\omega]_{\omega=\omega_{H,L}}}$$

is the linear damping rate of excited modes. It is evident from (28) that under the above assumptions and $\Omega \sim 2\omega_L$, the driver pump with frequency ω_0 contributes to the excitation of the high-frequency mode at ($\omega_1 - \omega_L,\mathbf{k}$) and the low-frequency mode at ($\omega_L,\mathbf{k}$). If however $\Omega = \omega_0 - \omega_1 \gg 2\omega_L$ (in Refs. 13 and 14, $\Omega > 5\omega_L$) the two resonances are decoupled and the sidebands ($\omega_1 \pm \omega_L,\mathbf{k}$), ($\omega_0 \pm \omega_L,\mathbf{k}$) do not mutually interact. (This is the case of separate action of two driver pumps with plasma.) In (28) interaction between sidebands leads to the energy transfer from the inner sideband ($\omega_0 - \omega_L,\mathbf{k}$) to the outer sideband ($\omega_1 - \omega_L,\mathbf{k}$) (see Refs. 13 and 14). As is seen from (28) the growth rate for the pump intensities above threshold is significantly larger than in the case of a single driver pump.

Let us now derive the explicit form of the influence of the $\Upsilon_+\Upsilon_-$ term on parametric excitations. From (16) we obtain

$$\left[R_e D(\omega^{(0)},\mathbf{k}) + \omega^{(1)}\left(\frac{\partial R_e D}{\partial\omega}\right)_{\omega=\omega^{(0)}} + iI_m D(\omega^{(0)},\mathbf{k})\right]\left[R_e D(\omega^{(0)}-\Omega,\mathbf{k}) + \omega^{(1)}\left(\frac{\partial R_e D}{\partial\omega}\right)_{\omega=\omega^{(0)}-\Omega} + iI_m D(\omega^{(0)}-\Omega,\mathbf{k})\right] = \Upsilon_+(-\omega_L,\mathbf{k})\Upsilon_-(\omega_L,\mathbf{k}). \quad (29)$$

Here $\omega^{(0)}$ is defined by $R_e D(\omega^{(0)}-\Omega,\mathbf{k}) \sim R_e D(\omega^{(0)},\mathbf{k}) = 0$. Consequently from (29) ($\gamma_{\mathrm{TOT}} = \gamma + \Delta\gamma$),

$$\Delta\gamma = \frac{1}{\gamma_1+\gamma_2}\Upsilon_+(-\omega_L,\mathbf{k})\Upsilon_-(\omega_L,\mathbf{k}) - \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \quad (30)$$

is obtained. Here $\Delta\gamma$ is defined by $\omega^{(1)} = R_e\omega^{(1)} + i\Delta\gamma$. $R_e\omega^{(1)}$ gives the linear frequency shift of parametrically excited modes and in (29) was neglected. γ_1 and γ_2 are given by

$$\gamma_1 = \frac{I_m D(\omega^{(0)},\mathbf{k})}{(\partial R_e D/\partial\omega)_{\omega=\omega^{(0)}}}, \quad \gamma_2 = \frac{I_m D(\omega^{(0)}-\Omega,\mathbf{k})}{(\partial R_e D/\partial\omega)_{\omega=\omega^{(0)}-\Omega}}, \quad (31)$$

where $\Delta\gamma$ represents the quantity which is for the order of magnitude smaller than γ and in the following it will be neglected.

As a concrete example of double resonance parametric excitations we shall study excitations of oblique-Langmuir waves $\omega_1 = \omega_{pe}|\cos\theta|$ coupled to ion-acoustic waves $\omega = \omega_s = kv_s|\cos\theta|$ and, excitation of electron-Bernstein modes coupled to ion-Bernstein modes. In the first case we have

$$\gamma = \frac{1}{16}\left(1 + \frac{1}{k^2 r_{De}^2}\right)^2 \sum_{n=1}^{2} (\mu^{(n)})^2 \omega_{pe} |\cos\theta| \times \left(\frac{\omega_s}{\omega_{pi}}\right)^2 \frac{1}{|\cos\theta|^2}. \quad (32)$$

In (32) it was taken into account that an ion-acoustic wave is heavily damped in ionospheric plasma due to $T_e \sim T_i$ $[\gamma_s \sim \omega_s = kv_s |\cos\theta|,\ v_s = (T_e/m_i)^{1/2}]$. Coupling coefficient $\mu^{(n)}$ has the following forms in the case of ordinary $(O)\mathbf{E}_n \| \mathbf{B}_0$ and, extraordinary $(X)\mathbf{E}_n \perp \mathbf{B}_0$ driver pumps, respectively,

$$\mu^{(n)}(O) = \frac{ekE_n}{m_e \omega_n^2} \cos\theta, \quad (33a)$$

$$\mu^{n}(X) = \frac{ekE_n}{m_e \omega_n^2}\left(\frac{\omega_n}{\Omega_e}\right)^2 \sin\theta. \quad (33b)$$

The parametric growth rate for the excitation of an ordinary Langmuir wave (treated in Ref. 4) could be obtained from (32) by putting $\cos\theta \equiv 1$. In this case [see (33b)] coupling of waves through X driver pumps is not possible.

The growing of electron-Bernstein $\omega = n|\Omega_e|$ and ion-Bernstein $\omega = m\Omega_i$ modes is defined by

$$\gamma^2 = \frac{1}{4}\left(1 + \frac{1}{k^2 r_{De}^2}\right)^2 \sum_{n=1}^{2} (\mu^{(n)})^2 \times \frac{(k^2\rho_i^2)^{m+1}(k^2\rho_e^2)^{n+1}(nm)(\Omega_e\Omega_i)^3}{(k^4 r_{Di}^4)(k^4 r_{De}^4)(\omega_{pi}\omega_{pe})^2 n! m! 2^{m+n}}. \quad (34)$$

In this case coupling is possible only by X driver pumps [in (33b)$\sin\theta \equiv 1$]. In (34) it was taken into account that linear dissipation of Bernstein modes is pure collisional and very small (typically for the F ionospheric layer $\nu_{ei}/\Omega_e < 10^{-3}$, where ν_{ei} is the electron–ion collision frequency). Due to this, threshold is lower and consequently channel (34) is more promising than (32) though it has not yet been experimentally studied. In a long-wavelength region $(k^2\rho_e^2 \ll 1)$ an electron-Bernstein mode $(n = 1)$ reduces to an upper-hybrid wave. Excitation of upper-hybrid waves was studied in the ionospheric simulation experiment in Ref. 15.

Finally, let us note that in experiments performed by Showen *et al.*[16], it was observed that a significant role is played by the so-called arithmetic mean instability in which high-frequency plasma response is at frequency $(\omega_0 + \omega_1)/2$. The growth rate for this instability could be obtained from (32) by putting, instead of $\omega_{pe}|\cos\theta|$, arithmetic mean frequency $(\omega_0 + \omega_1)/2$ and, the threshold value by substitution $\gamma \rightarrow \gamma_H$ ($\gamma_H \sim \nu_{ei}/2$ is the linear damping rate of oblique-Langmuir wave). The threshold value calculation for the arithmetic mean instability based on energy balance is given in Ref. 17.

VI. DISCUSSION AND CONCLUSION

In this paper a theory of parametric interaction of a multiple driver pump with magnetized plasma is developed. The theory is based on the Vlasov–Poisson formalism so that it represents the most general approach to the problem. Other approaches based on hydrodynamical equations (Refs. 4, 7, and 18 and Ref. 6 of Ref. 8) and energy-balance equations (Refs. 6 and 17) appear to be limiting cases valid only under certain circumstances. For example, the results of the hydrodynamical approach (Nishikawa's formalism) could be obtained from the Vlasov–Poisson formalism by taking plasma dielectric permittivities in hydrodynamical approximation and coefficients of interaction in weak driver-pump approximation. The limitation of the energy balance formalism is that coupling of certain modes must be known *a priori*, which is not always obvious.

It is shown that nonresonant interaction of a multiple driver pump with magnetized plasma leads to the appearance of new instabilities which are absent in the case of a single driver pump. A similar effect was shown in Part I with modulated driver pump. Here, we shall generalize this result saying that any kind of driver-pump nonmonochromaticity will lead in nonresonant interaction to appearance of instabilities (this kind of instability was first discovered by Aliev *et al.*, Refs. 3 and 12 and Ref. 22 of Ref. 8). Due to these instabilities (though the thresholds are higher for the order of magnitude with respect to the resonant parametric instabilities) the region of parametric excitations in ionospheric experiments could be significantly larger.

In resonant interaction it was shown that a number of sidebands appear which are absent in the case of a single driver pump. Furthermore the energy is transferred from one sideband to another (this is due to the nonlinear interaction of sideband lines if they are spaced at $\omega \sim 2\omega_L$). In the opposite case driver pumps are independent, and in the stationary phase, depending on the concrete experiment,[13,16,19] side-band structure is very complex. In our calculation we have utilized the results of experiments[13,14,15] to solve an algebraically very complex dispersion relation (16). Consequently the values of parametric growth rates and thresholds obtained in this paper are somewhat higher and lower, respectively, than in real situations where excitation of all side bands (with equal parametric growth rate) takes place. It is to be noted that our calculations predict very efficient excitation of long- and short-wavelength Bernstein modes by an X-mode driver pump if inherent problems in propagation of this mode through the ionosphere are avoided (see Ref. 19).

The theory developed is also applicable for electromagnetic wave generators which inherently appear to be multiple line generators like the HF laser.

In conclusion, Ref. 8 and the present paper give a linear theory of parametric excitations in magnetized plasma by a nonmonochromatic driver pump in a rather formal manner. Further detailed computational[20] and analytical consideration is needed in order that the results can be directly applied in practice. A natural continuation of the work is the development of a theory for the saturation processes of excited modes in the field of a nonmonochromatic driver pump. Some work has already been done in this area for isotropic plasma in Refs. 16 and 17 (Ref. 8).

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APPENDIX: DERIVATION OF MODEL EQUATIONS

Dispersion features of a magnetized, homogeneous plasma in the electric field of a multiple driver pump in the form $\mathbf{E}(t) = \Sigma_{n=0}^{N} \mathbf{E}_n \sin(\omega_n t + \varphi_n)$ is obtained utilizing a Vlasov–Poisson system of equations. The space-time evolution of a one-particle plasma distribution function $f_\alpha(\mathbf{r},\mathbf{p},t)$ is governed by

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\nabla_{\mathbf{v}} f_\alpha + e_\alpha\left(\mathbf{E}(t) + \mathbf{E}(r,t) + \frac{1}{c}(\mathbf{v}\times\mathbf{B})\right) \times \nabla_{\mathbf{p}} f_\alpha = 0, \quad \alpha = e, i. \tag{A1}$$

Here $\mathbf{E}(\mathbf{r},t)$ denotes a plasma self-cohsistent electric field which is obtained from the Poisson equation. In a perturbed state (denoted by superscript "1") the system (A1) obtains the following form:

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + \mathbf{v}\nabla_{\mathbf{v}} f_\alpha^{(1)} e_\alpha \mathbf{E}(t)\nabla_{\mathbf{p}} f_\alpha^{(1)} + \frac{e_\alpha}{c}(\mathbf{v}\times\mathbf{B}_0)\nabla_{\mathbf{p}} f_\alpha^{(1)} + e_\alpha \mathbf{E}^{(1)}(\mathbf{r},t)\nabla_{\mathbf{p}} f_\alpha^{(0)} = 0. \tag{A2}$$

Applying a Fourier transform in $\mathbf{r}$ space on system (A2) we obtain

$$\frac{\partial f_\alpha^{(1)}}{\partial t} + i\mathbf{k}\mathbf{v} f_\alpha^{(1)} + e_\alpha \mathbf{E}(t)\nabla_{\mathbf{p}} f_\alpha^{(1)} + \frac{e_\alpha}{c}(\mathbf{v}\times\mathbf{B}_0)\nabla_{\mathbf{p}} f_\alpha^{(1)} = i\mathbf{k}\nabla_{\mathbf{p}} f_\alpha^{(0)} \sum_\beta \frac{4\pi e_\alpha e_\beta}{k^2}\int f_\beta^{(1)}\,d\mathbf{p}. \tag{A3}$$

Here $f_\alpha^{(0)}$ is a nonperturbed one-particle distribution function. In (A3) we substituted a self-consistent plasma electric field

$$\mathbf{E}^{(1)}(\mathbf{r},t) = -\frac{4\pi i\mathbf{k}}{k^2}\sum_\beta e_\beta \int f_\beta^{(1)}(\mathbf{p},\mathbf{k},t)d\mathbf{p}. \tag{A4}$$

Let us now introduce a new function:

$$\Psi_\alpha(\mathbf{p},t) = f_\alpha^{(1)}\left(\mathbf{p}_\alpha + \sum_{n=0}^{N}\delta p_\alpha^{(n)}, t\right)\prod_{n=0}^{N}\exp(i\mathbf{k}\delta\mathbf{r}_\alpha^{(n)}), \tag{A5}$$

where

$$\frac{d^2}{dt^2}(\delta\mathbf{r}_\alpha^{(n)}) = \frac{e_\alpha}{m_\alpha}\mathbf{E}(t) + \frac{e_\alpha}{m_\alpha c}(\mathbf{v}\times\mathbf{B}_0), \tag{A6}$$

$$\delta\mathbf{r}_\alpha^{(n)} \Rightarrow \frac{1}{m_\alpha}\int_{-\infty}^{t} dt'\,\delta\mathbf{p}_\alpha^{(n)}(t'). \tag{A7}$$

As can be seen from (A5), function $\Psi_\alpha(\mathbf{p},t)$ represents the distribution function of the α plasma component in the frame oscillating with α particles. Oscillation of α particles in a multiple driver-pump electric field and a constant magnetic field $\mathbf{B}_0$ is described by Eqs. (A6) and (A7). Substituting (A5) into (A2) we obtain the following equation:

$$\frac{\partial \Psi_\alpha}{\partial t} + i\mathbf{k}\mathbf{v}\Psi_\alpha + \frac{e_\alpha}{c}(\mathbf{v}\times\mathbf{B}_0)\nabla_{\mathbf{p}}\Psi_\alpha = i\mathbf{k}\nabla_{\mathbf{p}} f_\alpha^{(0)}\sum_\beta\frac{4\pi e_\alpha e_\beta}{k^2}\int\Psi_\beta\,d\mathbf{p} \times\prod_{n=0}^{N}\exp(i\mathbf{k}\delta\mathbf{r}_\alpha^{(n)})\prod_{n=0}^{N}\exp(i\mathbf{k}\delta\mathbf{r}_\beta^{(n)}). \tag{A8}$$

Further we can write

$$\prod_{n=0}^{N}\exp(i\mathbf{k}\delta\mathbf{r}_\alpha^{(n)})\prod_{n=0}^{N}\exp(i\mathbf{k}\delta\mathbf{r}_\beta^{(n)}) = \prod_{n=0}^{N}\exp(-i\mu^{(n)}\sin\omega_n t + i\varphi_n) = \prod_{n=0}^{N}\sum_{l_n=-\infty}^{+\infty} J_{-l_n}(\mu^{(n)})\exp(il_n\omega_n t + il_n\varphi_n). \tag{A9}$$

In (A9) the coupling coefficient is given by (6) (Sec. II). Substituting (A5) in (A8) and applying the Fourier transform in time we obtain

$$-i\omega\Psi_\alpha + i\mathbf{k}\mathbf{v}\Psi_\alpha + \frac{e_\alpha}{c}(\mathbf{v}\times\mathbf{B}_0)\nabla_{\mathbf{p}}\Psi_\alpha = i\mathbf{k}\nabla_{\mathbf{p}} f_\alpha^{(0)}\sum_\beta\frac{4\pi e_\alpha e_\beta}{k^2}\int d\mathbf{p} \times\sum_{l_n=-\infty}^{+\infty}\prod_{n=0}^{N}J_{-l_n}(\mu^{(n)})e^{il_n\varphi_n} \times\Psi_\beta\left(\omega + \sum_{n=0}^{N} l_n\omega_n, \mathbf{k}\right). \tag{A10}$$

Here $\Psi_\alpha \equiv \Psi_\alpha(\mathbf{p},\mathbf{k},\omega)$. Writing (A10) for a two-component (electron–ion, $\alpha \equiv e, i$) plasma we obtain the following system of equations:

$$-i\omega\Psi_e + i\mathbf{k}\mathbf{v}\Psi_e + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_0)\nabla_{\mathbf{p}}\Psi_e = i\mathbf{k}\nabla_{\mathbf{p}} f_e^{(0)}\left(\frac{4\pi e^2}{k^2}\int\Psi_e\,d\mathbf{p} + \frac{4\pi e e_i}{k^2}\int d\mathbf{p} \times\sum_{l_n=-\infty}^{+\infty}\prod_{n=0}^{N}J_{-l_n}(\mu^{(n)})e^{il_n\varphi_n}\Psi_i\right), \tag{A11}$$

$$-i\omega\Psi_i + i\mathbf{k}\mathbf{v}\Psi_i + \frac{e_i}{c}(\mathbf{v}\times\mathbf{B}_0)\nabla_{\mathbf{p}}\Psi_i = i\mathbf{k}\nabla_{\mathbf{p}} f_i^{(0)}\left(\frac{4\pi e_i^2}{k^2}\int\Psi_i d\mathbf{p} + \frac{4\pi e e_i}{k^2}\int d\mathbf{p} \times\sum_{l_n=-\infty}^{+\infty}\prod_{n=0}^{N}J_{l_n}(\mu^{(n)})e^{il_n\varphi_n}\Psi_e\right). \tag{A12}$$

In (A11) and (A12) $\mu^{(n)} \equiv \mu_{ei}^{(n)}$. Introducing the charge density of the α plasma component in an oscillating frame,

$$n_\alpha(\omega,\mathbf{k}) = e_\alpha\int d\mathbf{p}\,\Psi_\alpha(\mathbf{p},\mathbf{k},\omega), \tag{A13}$$

and using a known procedure[9] of the plasma dielectric permittivity formalism, we obtain

$$n_e(\omega,\mathbf{k}) + \chi_e(\omega,\mathbf{k})n_e(\omega,\mathbf{k}) = -\chi_e(\omega,\mathbf{k})\sum_{\substack{l_n=-\infty \\ n=0,1\ldots N}}^{+\infty}\prod_{n=0}^{N}J_{-l_n}(\mu^{(n)})e^{il_n\varphi_n} \times n_i\left(\omega + \sum_{n=0}^{N} l_n\omega_n, \mathbf{k}\right), \tag{A14}$$

$$n_i(\omega,\mathbf{k}) + \chi_i(\omega,\mathbf{k})n_i(\omega,\mathbf{k})$$
$$= -\chi_i(\omega,\mathbf{k}) \sum_{\substack{l_n=-\infty \\ n=0,1\ldots N}}^{+\infty} \prod_{n=0}^{N} J_{l_n}(\mu^{(n)})e^{il_n\varphi_n} \times n_e\Big(\omega + \sum_{n=0}^{N} l_n\omega_n,\mathbf{k}\Big). \tag{A15}$$

Here $\chi_\alpha(\omega,\mathbf{k})$ is the linear susceptibility of the α plasma component given by (4) (Sec. II). With quantity R_α defined by $R_\alpha = \chi_\alpha(\omega,\mathbf{k})[1+\chi_\alpha(\omega,\mathbf{k})]^{-1}$ from (A14) and (A15), we obtain:

$$n_e(\omega,\mathbf{k}) = -R_e(\omega,\mathbf{k}) \sum_{\substack{l_n=-\infty \\ n=0,1\ldots N}}^{+\infty} \prod_{n=0}^{N} J_{-l_n}(\mu^{(n)}) \times e^{il_n\varphi_n} n_i\Big(\omega + \sum_{n=0}^{N} l_n\omega_n,\mathbf{k}\Big), \tag{A16}$$

$$n_i(\omega,\mathbf{k}) = -R_i(\omega,\mathbf{k}) \sum_{\substack{l_n=-\infty \\ n=0,1\ldots N}}^{+\infty} \prod_{n=0}^{N} J_{l_n}(\mu^{(n)}) \times e^{il_n\varphi_n} n_e\Big(\omega + \sum_{n=0}^{N} l_n\omega_n,\mathbf{k}\Big). \tag{A17}$$

In the case of an isotropic plasma (see Ref. 3) the system (A16) and (A17) is also valid but with

$$\chi_\alpha(\omega,\mathbf{k}) = \frac{4\pi e_\alpha^2}{k^2}\int \frac{d\mathbf{p}\,\mathbf{k}\nabla_\mathbf{p} f_\alpha^{(0)}}{\omega - \mathbf{kv}}, \quad \mu^{(n)} = \frac{e\mathbf{k}\mathbf{E}_n}{m_e\omega_n^2}. \tag{A18}$$

The system (A16) and (A17) could be very easily reduced to the case of a single[9] driver-pump parametric theory. In doing so let us formally write $\omega \to \omega + s\omega_0$. Then from (A16) and (A17) we obtain

$$n_e^{(s)} = -R_e^{(s)} \sum_{\substack{m,l_n=-\infty \\ n=1\cdots N}}^{+\infty} J_{s-m}(\mu) \prod_{n=1}^{N} J_{-l_n}(\mu^{(n)}) \times n_i\Big(\omega + m\omega_0 + \sum_{n=1}^{N} l_n\omega_n,\mathbf{k}\Big)e^{il_n\varphi_n}, \tag{A19}$$

$$n_i^{(s)} = -R_i^{(s)} \sum_{\substack{m,l_n=-\infty \\ n=1\cdots N}}^{+\infty} J_{m-s}(\mu) \prod_{n=1}^{N} J_{l_n}(\mu^{(n)}) \times n_e\Big(\omega + m\omega_0 + \sum_{n=1}^{N} l_n\omega_n,\mathbf{k}\Big)e^{il_n\varphi_n}, \tag{A20}$$

where

$$R_\alpha^{(s)} = \chi_\alpha(\omega + s\omega_0,\mathbf{k})[1 + \chi_\alpha(\omega + s\omega_0,\mathbf{k})]^{-1}.$$

If $E_n \equiv 0$ for $n = 1\ldots N$ then $\mu^{(n)} = 0$. Consequently

$$J_{l_n}(\mu^{(n)}) = \begin{cases} 1, & l_1,l_2\ldots l_N = 0, \\ 0, & l_1,l_2\ldots l_N \neq 0, \end{cases}$$

and

$$\sum_{n=1}^{N} l_n\omega_n = 0.$$

Now, the system (A19) and (A20) is reduced to the model equations of a single driver-pump parametric theory[9] ($\mu^{(0)} \equiv \mu$).

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