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V. Alexander Stefan

The ROSENBLUTH Convective Parametric Amplification of Plasma Waves

The Role of Parametric Plasma Turbulence in the Laser Thermonuclear Fusion Research

Dedicated to the Memory of

MARSHALL NICHOLAS ROSENBLUTH

(February 5, 1927, Albany, New York -
September 28, 2003, La Jolla, California)

Marshall Nicholas Rosenbluth
(1927—2003)
February 1992, La Jolla, California.

Courtesy of the Institute for Advanced Physics Studies,
La Jolla, California.

1. The Rosenbluth Convective Parametric Amplification of Plasma Waves

In the 1972 paper [1], Marshall Rosenbluth developed the theory of the three-wave parametric coupling for weakly inhomogeneous media. He studied the two plasmon parametric excitation in the WKB approximation. The plasma inhomogeneity is assumed to be linear:

$$\omega_{pe}^{2}(x) = \omega_{pe}^{2}(x_c)\,(1+x/L_N), \qquad (1)$$

$$L_N = (d \ln n_e\,(x)/\,dx)^{-1}, \qquad (2)$$

$$\omega_{pe}^{2}(x) = 4\pi\, n_e(x)\, e^2/\, m_e. \qquad (3)$$

Here $\omega_{pe}(x)$ is the angular electron plasma frequency; $x_c=0$ is the location of the crirtical surface defined by: $\omega_0 = \omega_{pe}\,(x_c)$; ω_0 is the angular frequency of the laser radiation; $n_e\,(x)$ is the plasma linear density profile; L_N is the plasma inhomogeneity scale length; e and m_e are the electron charge and mass, respectively.

In the approximation of a homogeneous laser radiation electric field, it is shown by Rosenbluth that the two-plasmon parametric decay, which takes place in the quarter critical density of laser plasma corona, $\omega_0 = 2\omega_{pe}(x_{c/4})$, has a convective character. As a consequence, the excited plasma waves are convectively amplified. The convective parametric amplification is also studied by A.A. Galleev et. al., (1973) [3].

In addition to the convective parametric amplification of plasma waves, Rosenbluth carried out a seminal research in the generation of laser beat waves. He was among the first to address the importance of plasma density inhomogeneity effects on parametrically excited plasma waves.

In 1973, Rosenbluth and Sagdeev studied various parametric instabilities, focusing on the application to laser thermonuclear fusion [2] The Rosenbluth parametric amplification has been used in the theory of free electron lasers and plasma electron acceleration, {see for example a review by Stefan, Cohen, and Joshi (1989) [17.c.]}

The philosophy on how to achieve the optimal laser pellet coupling has been changing through the years. In this paper I have in mind the ideas and laser thermonuclear fusion experiments of the 1970s. *Accordingly, I discuss some major work on the convective-absolute character of parametric instabilities in the research centers in the States, (with the focus on the work of Rosenbluth with his collaborators), and in Russia in the 1970s* [4].

The presentation style in the seminal paper [1] is typical of Rosenbluth. Here is what Freeman Dyson[1] says about the Rosenbluth style,

"*Rosenbluth's papers were written in the Fermi style, cutting out inessential details and making the main points easy to understand.*"

A 1972 paper of Rosenbluth initiated a vigorous research in the character of parametric instabilities, convective versus absolute, in a laser, [see Fig.1], Tokamak, magnetic mirror, and bumpy torus thermonuclear fusion research {Brueckner and Jorna

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[1] Freeman Dyson (b. 1923), a British-born American physicist.

(1974)[5]}, Afanasiev et al., (1978) [6], Nuckolls et. al. (1972) [7], Kidder (1991) [8], Brueckner (1992) [9], Rosenbluth (1994) [10], the work at various laboratories in the USA and Russia [11]}.

In Russia,[2] they say: when the Tsar builds there is a plenty of work for the masons' guilds. I see the Rosenbluth 1972-paper in that spirit. It is rich in ideas, of a deep scope, and error free. In his 1955-*Science* paper [12], Edward Teller[3] says that many of his young collaborators have made errors, but Rosenbluth's calculation was error-free.

2. The "Gradient Effects" on Convective-Absolute Character of Parametric Plasma Instabilities

The interplay between the plasma density inhomogeneity, the inhomogeneity of plasma electron temperature, and the inhomogeneity of the laser radiation electric field affects crucially the character of plasma parametric instabilities: the convective versus absolute. In this paper the phenomena due to the gradients are referred to as the "Gradient Effects." The gradients can cause the convective-absolute interchange in the character of parametric instabilities. In the work by Piliya [13], whereby the homogeneous electric field is assumed, it is shown

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[2] Marshall Rosenbluth's ancestors moved to the States from Odessa, Russia, at the turn of the 20th century.

[3] Edward Teller (1908—2003), a Hungarian born American physicist, known as "the father of the American hydrogen (thermonuclear) bomb."

that the temperature gradient can lead to the change of the convective into the absolute instability.

The plasma density inhomogeneity causes a variety of effects: it narrows the parametric resonance region, which enhances the parametric threshold values; it causes the appearance of the reflection surfaces in plasmas and, as a consequence, the possibility for the counter propagation of the parametrically coupled plasma modes. This, in turn, can lead to the appearance of the absolute parametric instabilities as shown by Perkins and Flick [14].

3. Parametric Thresholds in Inhomogeneous Plasma

A significant increase of the parametric threshold value in inhomogeneous plasma occurs if: $l_{ei} > L_N$, where l_{ei} is the electron mean free path, $l_{ei} = v_{Te} / \nu_{ei}$, v_{Te} is the electron thermal velocity. This is natural if we are reminded of the fact that the strong parametric coupling, as a rule, is realized through the dynamics of the electron plasma component.

The narrowing of the region of the parametric excitations due to the plasma inhomogeneity is described by, {[1] and [2]}:

$$\int_0^{\Delta x_T} (\mathbf{k}_0 - \mathbf{k}_1 - \mathbf{k}_2)\, dx = 1 \qquad (4)$$

Δx_T is the width of the region of parametric coupling and approximately the width of the parametric turbulence region..

4. Parametric Interaction of Laser Radiation with Inhomogeneous Plasma

The propagation of electromagnetic wave in inhomogeneous plasma is studied by Ginzburg [16]. The Ginzburg profile of the laser electric field for the case of normal laser incidence is given by:

$$E_0(x) = 2 E_0 (k_0 L_N)^{1/6} A_i \{(k_0 L_N)^{2/3} [(x/L_N)-\nu_{eff}/\omega_0]\}. \qquad (5)$$

In (5), $E_0(x)$ is the laser electric field profile; A_i denotes the Airy function, which is oscillating for $x < 0$. The width of the Airy function maximums is given by $(c^2 L_N/\omega_0^2).^{1/3}$ It is also approximately the distance between them. The wave number of the laser radiation is k_0. The effective collision frequency, ν_{eff}, which describes a variety of the plasmon dissipation precesses, is given by:

$$\nu_{eff} = \max\{\nu_{ei}; \omega_{pe}(r_{De}/L_N)^{2/3}; (\omega_0 V_p/L_N)^{1/2}; \nu_T\} \qquad (6)$$

In (6), the second term describes the dissipation of plasmons due to the convection; r_{De} is the electron Debye length. The third term describes linear saturation mechanism due to the plasma corona expansion; V_p is the expansion velocity. The parametric turbulence collision frequency, $\nu_T(E_0)$, due to the weak and strong parametric turbulence is discussed by Rosenbluth and Sagdeev [2], and due to the generation of suprathermal electrons, quasistationary magnetic fields, and laser radiation harmonics by Stefan [17b].

In the case of normal incidence, laser radiation does not reach the critical surface. For the oblique incidence, however, the tunneling effect takes place in the region of the reflection surface, so that the electric field structure is built up in the area of the critical surface. The field structure is given by, Ginzburg (1964) [16]:

$$E_{0,max}(x) = E_0 [\Phi(\theta_0)_{max}/2\pi k_0 L_N)^{1/2} (\omega_0/\nu_{eff}) \qquad (7)$$

Here $\Phi(\theta_0)$ is the Ginzburg function: $\Phi(\theta_0)_{max} \sim 1.2$ for $\theta_{0,\ max} = 0.7$; θ_0 is the angle of incidence.

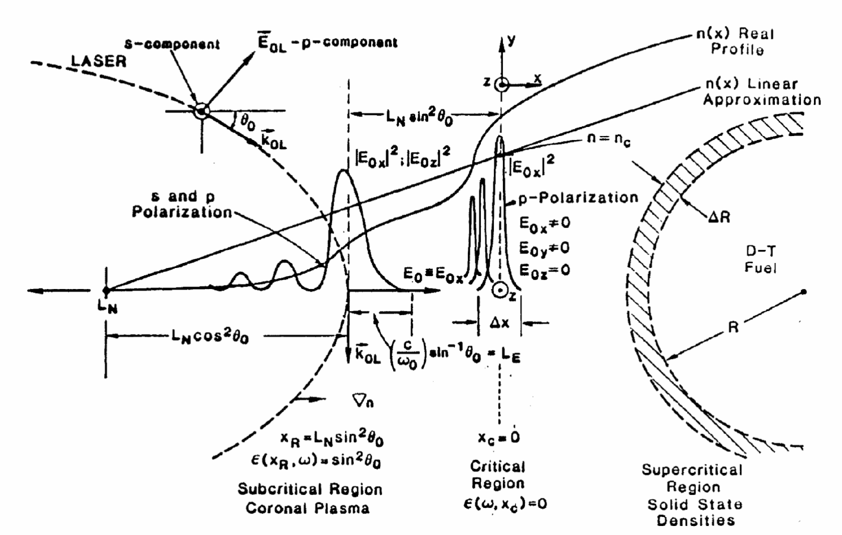

Figure1. The geometry of laser radiation thermonuclear fusion pellet interaction. Laser radiation electric field has two components: p-component in the (x,y) plane and s-component, which is normal to the (x,y) plane. The formation of plasma is assumed to be 1D with inhomogeneity along the x-axis and inhomogeneity scale length L_N. θ_0 is the angle of incidence and L_E the width of the evanescence of laser electric field. The critical region is defined by $\varepsilon(\omega, x_c) = 0$, ($\varepsilon$ is plasma dielectric permittivity), The reflection region is defined by $\varepsilon(\omega, x_R) = \sin^2\theta_0$. ΔR is the thickness of the thermonuclear fusion pellet shell and R is its radius. The structure of the electric field in the reflection region is qualitatively the same for the s- ($E_{0x} = E_{0y} = 0$, $E_{0z} \neq 0$) and p-polarization (E_{0x} ; $E_{0y} \neq 0$, $E_{0z} = 0$). The structure of the laser field in the critical region is shown for the case when nonlinear processes occur, leading to the appearance of a multiple-peak structure of the laser electric field.

The propagation of laser radiation through the inhomogeneous plasma leads to the variety of parametric processes:

a). Two plasmon decay: $t \rightarrow l + l$, $\omega_0 \sim 2\omega_{pe}$. Here, t denotes laser radiation, a transverse wave; l denotes a plasmon, (a longitudinal wave - an electron Langmiur plasma mode).

b). Stimulated Raman scattering: $t \rightarrow t + l$, $\omega_0 \geq 2\omega_{pe}$.

c). Plasmon-phonon decay: $t \rightarrow l + s$, $\omega_0 \sim 2\omega_{pe}$. Here s denotes a phonon, an ion sound (acoustic) plasma mode.

d). Stimulated Brillouin scattering: $t \rightarrow l + s$, $\omega_0 \geq \omega_{pe}$.

e) The secondary parametric decays can occur in both critical and quarter critical density regions. The excited plasmons can further decay, (secondary decays), into another plasmon, l_1, and a phonon, $l \rightarrow l_1 + s_1$, and so on. This process is referred to as the plasmon cascading. Another possibility for a secondary decay is: $l \rightarrow l_1 + a$. Here, "a" denotes the aperiodic low frequency mode. If the low frequency mode is heavily damped, the process is referred to as the stimulated plasmon scattering on ions, $l \rightarrow l_1 + i$. The plasmons can also be scattered off electrons, $l \rightarrow l_1 + e$.

f). Modulational instability can occur in both critical and quarter critical density regions. The growth of plasmon can trigger modulational instability and subsequent plasmon collapse {Zakharov (1972) [29]}

The laser electric field gradient in the regions of the Airy function maximums affects the parametrically driven plasma modes in a crucial manner: due to the electric field gradients, the wave number component along the gradient $k_x \rightarrow 0$. This means that the plasma mode(s) is "trapped" by the laser field gradient and, consequently, grows in time at that region. Accordingly, parametric coupling has an absolute character.

In the laser field given by (5) and (7), the convective-absolute interchange can occur. This is also true for the expanding laser plasma corona. Depending on the velocity of the plasma corona expansion, the absolute parametric instabilities can be converted into convective.

5. Two Plasmon and Plasmon–Phonon Decay: Dispersion Relations

The two plasmon parametric coupling in an inhomogeneous plasma (t → l + l) is described by the following dispersion relation, {Silin (1973) [24]; Kaw at al. (1976) [25]; Stefan, Krall, and McBride (1997) [17 a]}:

$$\varepsilon_l\,(\omega_1, \mathbf{k}_1, x)\;\; \varepsilon_l\,(\omega_1 - \omega_0, \mathbf{k}_1 - \mathbf{k}_0, x) = [(\mathbf{k}_1 - \mathbf{k}_0) \bullet \mathbf{r}_E]^2\; [(2\mathbf{k}_1 - \mathbf{k}_0) \bullet \mathbf{k}_0]^2 / k^2\, (\mathbf{k}_1 - \mathbf{k}_0)^2 \tag{8}$$

$$\omega_0 = \omega_1 + \omega_2 \tag{9}$$

$$\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2 \tag{10}$$

Dielectric permittivities of the parametrically excited plasmons are ε_l (ω_1, $\mathbf{k}$, x) and ε_l (ω_1- ω_0, $\mathbf{k}_1$- $\mathbf{k}_0$, x) {Krall and Trievelpice (1973) [18]}.

$$\mathbf{r}_E = e\, \mathbf{E}_0 / m_e\, \omega_0^2\,. \tag{11}$$

From (8), it follows that the parametrically excited plasmons counter propagate [18] in the plane normal to $\mathbf{k}_0$ x $\mathbf{E}_0$. The propagation of the plasmons takes place at the angle 45^o with respect to $\mathbf{E}_0$.

For the plasmon-phonon decay instability, the dispersion relation takes the form, {Silin (1973) [24.a]; Stefan, Krall, and McBride (1997) [17 a]}:

$$\varepsilon_s(\omega_s,\mathbf{k}_s,x)\varepsilon_l(\omega_s - \omega_0,\mathbf{k}_s - \mathbf{k}_0,x) =$$

$$k^2(\omega_{pe}/\omega_{pi})^2[(\mathbf{k}_s - \mathbf{k}_0) \bullet \mathbf{r}_E]^2/\mathbf{4}(\mathbf{k}_s - \mathbf{k}_0)^2 \tag{12}$$

Here $\varepsilon_s(\omega_s,\mathbf{k}_s,x)$ is the dielectric permittivity of ion-acoustic mode.

6. Stimulated Raman and Brillouin Scattering: Dispersion Relations

Dispersion relation for the stimulated Raman Scattering, t → t + l, $\omega_0 \geq 2\omega_{pe}(x)$, {Silin (1973) [24.a]; Kaw at al. (1976) [25]; Stefan, Krall, and McBride (1997) [17 a]; C.S.Liu and V.K.Tripathi 1994) [25.b]; W.L. Kruer (2003) [25.c]}, is:

$$\varepsilon_l(\omega_l,\mathbf{k}_l,x)+\{\mu^-/D_t^- + \mu^+/D_t^+\} = 0. \tag{13}$$

$$\mu^{+,-} = (1/4\}\ (k/\mathbf{k}_l \pm \mathbf{k}_0)^2[\omega_{pe}(x)/\omega]^2\ [(\mathbf{k}_l \pm \mathbf{k}_0)\ x\ \mathbf{r}_E]^2, \tag{14}$$

$$D_t^{+,-} = \varepsilon_t(\omega \pm \omega_0,\mathbf{k}_l \pm \mathbf{k}_0,x)-c^2(\mathbf{k} \pm \mathbf{k}_0)^2/(\omega_l \pm \omega_0)^2. \tag{15}$$

Stokes, $(^-)$, and anti Stokes, $(^+)$, components of the scattered laser radiation, $\mathbf{k}_t^{\pm} \bullet \mathbf{E}_t^{\pm} = 0$, according to the conservation of energy and impulse, satisfy:

$$D_t^{+,-} \sim 0. \tag{16}$$

The dielectric permittivities of the parametrically excited plasmon, $\varepsilon_l\ (\omega_l,\ \mathbf{k}_l,\ x)$, and scaterred laser radiation, $\varepsilon_t\ (\omega_l \pm \omega_0,\ \mathbf{k}_l \pm \mathbf{k}_0,\ x)$, are given by Krall and Trievelpice (1973) [18]. If the excited plasmon is heavily damped, $\omega_0 >> \omega_{pe}(x)$, by linear Landau damping, the dispersion relation (10) describes stimulated Compton scattering.

The dispersion relation for the stimulated Brillouin scaterring is given by (10), whereby $\varepsilon_l\ (\omega_l,\ \mathbf{k}_l,\ x) \longrightarrow \varepsilon_s\ (\omega_l,\ \mathbf{k}_l,\ x)$.

7. Two Plasmon and Plasmon–Phonon Decay: ParametricThresholds, Growth Rates, and Turbulence Collision Frequencies

For the absolute two plasmon decay: $\Delta x_T \sim L_N \ln^{-1} (\omega_{pe}/\nu_{ei})$, the parametric threshold in a high temperature inhomogeneous plasma, $l_{ei} >> L_N$, upon the action of the laser electric field (4),{Rosenbluth and Sagdeev (1973) [3}, {Silin and Starodub (1974) [15.a.]}, is given by:

$$(r_{E, THR} / r_{De}) \sim (\sqrt{3} /2\, k_0 L_N)^{1/2} . \qquad (17)$$

The parametric growth rate is given by:

$$\gamma_{l+l} = k_0 r_E\, \omega_{pe} \qquad (18)$$

In the paper by Silin and Starodub (1974) [15.a.], it is shown that in the electric field (4), two plasmon parametric decay is an absolute instability. The excited two plasmons counter propagate and “collide,” which in an inhomogeneous plasma lead to the plasmon wave number along the density gradient, $k_x \rightarrow 0$. Consequently, the two plasmons are trapped by the “collision.” The local turbulence level is built up and saturated, among other competing saturation mechanisms, via plasmon modulational instability {Rosenbluth and Sagdeev (1973) [3].

The trapping of the parametrically excited plasmons, $t \rightarrow l + l$, due to the plasma inhomogeneity is studied by Lee and Kaw (1974) [20]. It is shown that plasmon trapping prevents the wave energy convection from the coupling region. Consequently, the two plasmon decay is an absolute parametric instability.

In 1976, Liu and Rosenbluth [21] studied a high temperature inhomogeneous plasma, $\nu_{ei} \rightarrow 0$, in interaction with the laser radiation electric field (3), ($t \rightarrow l + l$. It is shown that the two-plasmon decay has an absolute character if: $kr_E < (3/2) (k\, r_{De}).^2$ If $k < k_{coll}$, this condition is always satisfied. Here, k_{coll} is the plasmon collisional wave number at which the plasmon linear Landau dissipation is equal to the collisional dissipation.

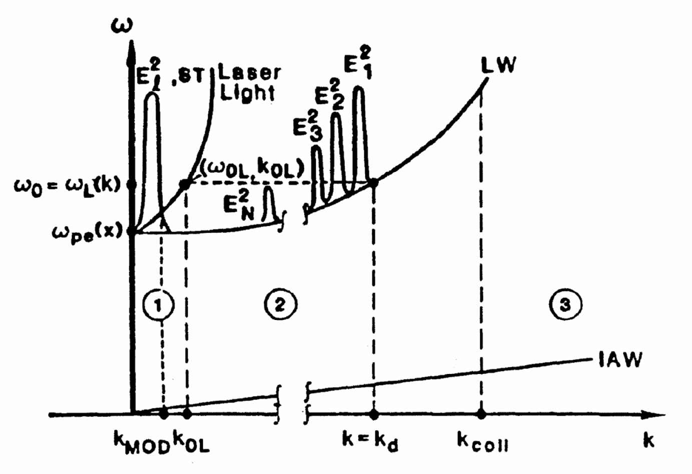

Figure 2. The dispersion curves for the laser light - a phonon, plasmon - a Langmuir plasma mode, and a phonon – an ion acoustic plasma mode. The region (1) corresponds to the excitation of the long wavelength plasmons – the strong parametric plasma turbulence region. The region (2) corresponds to the weak parametric plasma turbulence and the region (3) to the strong linear Landau damping. k_{coll} is the collisional wave number; k_d the decay wave number, and k_{MOD} the modulational wave number. Convection of linearly excited plasmons is mainly along the density gradient, while in the case of parametrically excited plasmons in the plane of incidence ($\mathbf{k}_0$, x). The plasmon cascading is presented in the form of the line-like satellites, which corresponds to a weak parametric coupling approximation.

The saturation of the two plasmon decay can be achieved via the cascading saturation mechanism {Rosenbluth and Sagdeev 1973 [2]; namely, via the secondary decays of plasmons: plasmon decays into another plasmon coupled to phonon (ion acoustic mode, or a heavily damped phonon, (this is the case of the stimulated plasmon scattering on ions){Silin (1973) [24], Kaw, Kruer, Liu, Nishikawa (1976) [26]}. Another possibility is the quasilinear plasmon-electron interaction{Galeev, Oraevsky, and Sagdeev (1972) [23].

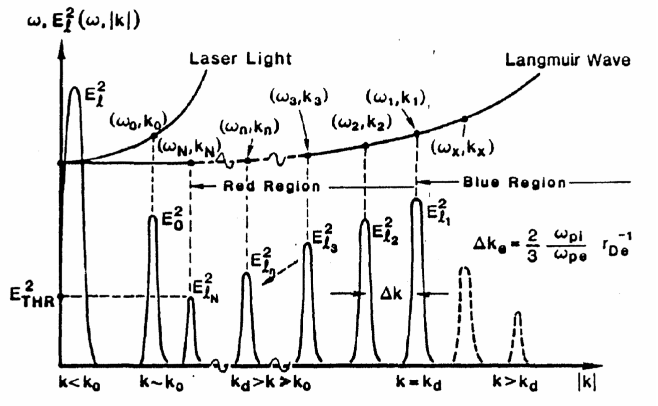

Figure 3. Strong parametric coupling of plasmons (Langmuir wave) and phonons (ion acoustic wave). In this case the turbulence spectrum in the "red" region ($k < k_d$) is continuous and accompanied with the "blue" satellites ($k > k_d$).

In the case of the cascading saturation mechanism,{Afanasiev et al. (1978) [6]; V. P. Silin and V. T. Tikhonchuk (1986) [24. B]}, the effective collision frequency is:

$$\nu_{T,\, l+l} = \nu_{ei} E_l^{\,2}/E_0^{\,2} = 2\,(\omega_{pe} / \omega_s)\,(v_{Te}/c)^2\,\gamma(E_o, L_N) \qquad (19)$$

Here, $\gamma(E_o, L_N)$ is the inhomogeneous plasma parametric growth rate for the secondary decay in the quarter critical density region {a detailed description is provided in: Afanasiev et. al. (1978) [6], p.129}. In (19), ω_s is the ion acoustic angular frequency. From (21) the stationary parametric plasma turbulence level, E_l^2, can be estimated.

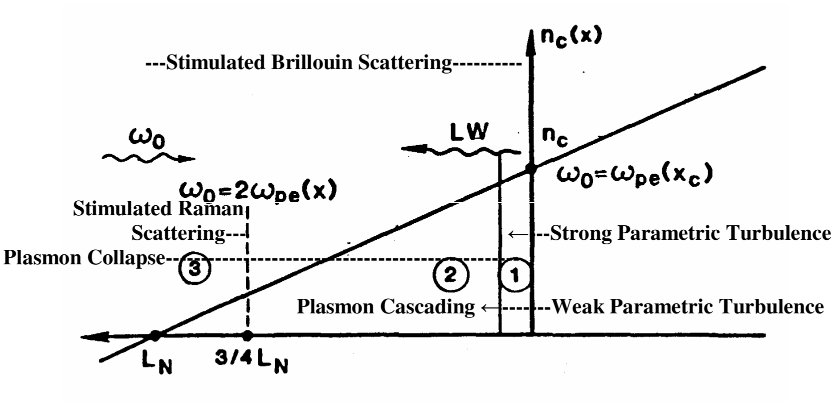

Figure 4. Parametric Plasma Turbulence Regions. LW – Langmuir wave, plasmon. Region (1) corresponds to the strong plasma parametric turbulence. Region (2): cascading of plasmons – weak plasma turbulence. In region (3), a strong linear Landau damping takes place. n_c is the critical plasma density.

Parametric turbulence collision frequency can be estimated based on other saturation mechanisms. Liu and Rosenbluth [21], Liu Rosenbluth, and White [23] studied the pump depletion as the saturation mechanism. The density profile steepening in the quarter critical density, as a saturation mechanism, was studied by Langdon et. al., (1979) [27]. Modulational instability and caviton formation could also be an efficient saturation mechanism [2] for the two plasmon parametric instability [28].

The plasmon-phonon decay can be either absolute or convective instability. The convective-absolute character of the decay instabilities in the critical region, ($t \rightarrow l + s$), is studied by Silin and Starodub, (1977) [15.b.]. It is shown that the parametric decay instability is absolute if the inhomogeneous character of the laser field is taken into account. In the paper of Perkins and Flick, it is shown that the aperiodic oscillating two stream instability, ($t \rightarrow l + a$), which occurs beyond the critical surface, is the absolute instability in the approximation of a homogeneous laser electric field.

In the case of absolute plasmon-phonon decay, $\Delta x_T << L_N$ and, accordingly, it is not important in view of the laser radioation absorption. It is quite different with the convective plasmon-phonon decay. Here $\Delta x_T \sim L_N$, so that the convective instability is preferential. The convective threshold is given by, Afanasiev et. al. (1978) [6]:

$$k_0 r_{E,\,THR} \sim (v_{Te}/c)\,(k_s L_N)^{-1/2} \qquad (20)$$

The growth rate is:

$$\gamma_{l+s} = k_0 r_E \ (c / v_{Te}) \ (\omega_0\, \omega_s)^{1/2} \qquad (21)$$

In the region defined by: $\omega_{pi}\,(m_e/ m_i)^{1/2} << \omega_0 - \omega_{pe}(x) << \omega_0 \ln^{-1}(\omega_0/\nu_{ei})$, the most favorable saturation mechanism is the plasmon cascading{see Kruer in [25.a]}. The turbulence collision frequency is given by:

$$\nu_{T,\, l+s} = 2\, \gamma_{l+s} \qquad (22)$$

In a strong turbulence region, Fig.2, 3, and 4, the modulational instability leads to a caviton formation with the trapped plasmons - Langmuir modes. Subsequently, the trapped plasmons collapse, transferring the energy into the domain of strong linear Landau damping. This is the mechanisms of the generation of suprathermal electron component in laser fusion plasma. The suprathermal electrons have a deteriorating effect: they lead to the preheat of the pellet core. The turbulence collision frequency is given by, {Afanasiev et. al. (1978) [6]}:

$$\nu_T^{\text{plasmon collapse}} = \omega_{pe}\,(m_i/ m_e)\ (k_0 r_E)^4\, x$$

$(\omega_0 / \omega_{pe})^4 (c/v_{Te})^4$ (23)

4.2. Stimulated Raman and Brillouin Scattering: Parametric Thresholds, Growth Rates, and Turbulence Collision Frequencies

The convective-absolute character of the stimulated Raman scattering is studied by Silin and Starodub (1974) [15.c.]. It is shown that the stimulated Raman scattering becomes an absolute parametric instability if the wave number of the scattered laser radiation, k_t,is smaller than k_0, which is satisfied in the vicinity of $n_c/4$, $\omega_0 \sim 2\omega_{pe}$. In this case the trapping of the scattered laser radiation takes place, leading to the absolute nature of the instability. For $\omega_0 >> 2\omega_{pe}(x)$, it follows that $k_t \sim k_0$, so that the stimulated Raman scattering is the convective instability.

The threshold value for the absolute Raman scattering, {Silin and Starodub (1974) [15.c.]}, is given by:

$k_0 \, r_{E,\,THR} \sim (k_0 L_N)^{-2/3}$ (24)

From (17) and (24) it follows that the absolute Raman threshold is higher than the threshold for the absolute two plasmon decay.

The growth rate is given by:

$\gamma_{t+l} = k_0 \, r_E \, (\omega_{pe} \, \omega_0)^{1/2}$ (25)

Stimulated Raman scattering, t → t + l, and two plasmon decay, t → l + l, are the competing parametric processes in the quarter critical density region {Rosenbluth, White, Liu (1973) [22}; Liu, Rosenbluth, White (1973) [23]; Silin and Starodub (1974) [15.c.]}. In the approximation of homogeneous plasma, the thresholds for the stimulated Raman scattering and two plasomon decay are identical:

$$k_0\, r_{E,\,THR} \sim (\nu_{ei} / \omega_0),\ \ k_0\, L_N \rightarrow 0 \qquad (26)$$

In the case of the laser electric field intensities much above the threshold value the growth rates of stimulated Raman and two plasmon instabilities are identical, {Rosenbluth and Sagdeev 1973 [3]}:

$$\gamma_{t+l} \sim k_0 r_E\, \omega_0\, ;\ (r_E / r_{E,\,THR}) = m >> 1. \qquad (27)$$

Large “overthreshold” coefficient, m >>1, indicates a strong coupling of the parametrically excited modes when their dispersion characteristics are significantly modified.

Saturation of the stimulated Raman scattering due to the quasilinear plamon-electron interaction is studied by Gallev, Oraevky, and Sagdeev (1972) [26], which seems to be adequate for the case of convective Raman instability.

In the case of absolute stimulated Raman scattering, Silin and Starodub [15.c.], the turbulence level of plasmons can be high, so as to trigger the modulational instabilities {Rosenbluth and Sagdeev [3] V.E Zakharov (1972) [29]}. Modulation instability, (strong plasma parametric turbulence, takes place for k, k_0, satisfying: $3\ k_0\ r_{De} < (m_e/m_i)^{1/2}$; $k < k_0$. A physical picture of the stimulated Raman scattering saturation via the strong turbulence process is given by Rosenbluth and Sagdeev. Based on their arguments, ν_T is given by:

$$\nu_{T,\,t+l} = (c / \lambda_0)\, (v_{Te} / c)^4 (\omega_{pe} / \omega_0)^2 \qquad (28)$$

A high turbulence collision frequency does not necessarily mean a high efficiency of laser absorption.

In the case of the stimulated Brillouin scattering, the minimum threshold is achieved if $k^2 = 2\mathbf{k}\bullet\mathbf{k}_0$ is satisfied. The convective threshold is given by, {Rosenbluth and Sagdeev (1973) [3]}:

$$k_0 r_E = (v_{Te}/ c) (k_0 L_N)^{-1/2} \quad (29)$$

The growth rate is:

$$\gamma_{t+s} \sim k_0 r_E (\omega_{pe} / \omega_0) (c / v_{Te}) (\omega_0 \omega_s) \quad (30)$$

The stimulated Brillouin scattering is not important parametric instability for the absorption of laser radiation in plasma corona.

6. Laser Radiation Absorption due to the Parametric Plasma Turbulence: Convective Versus Absolute Parametric Instabilities.

As to the absorption efficiency of laser radiation via the parametric turbulence, {Afanasiev et al., (1978) [6]}, it is to be noted:

- Low threshold for a particular parametric instability does not mean a higher importance from the aspect of laser radiation absorption. Instability with a higher threshold can have much faster growth rate.
- The absolute character of a particular parametric instability does not lead necesseraly to a higher laser absorption rate. In some cases, due to the wide region of coupling, Δx_T, convective parametric instabilities lead to higher radiation absorption.
- High parametric turbulence collision frequency still does not mean necessarily a high absorption of laser radiation.

To evaluate the relevance of a particular parametric process with respect to the laser radiation absorption, the following criteria should be considered:

$$(\nu_T/\nu_{ei})(\Delta x_T/L_N) = n > 1. \qquad (31)$$

$$(I_0 / I_{0,THR}) = m > 1. \qquad (32)$$

The demand that the parametric absorption coefficient is larger than the thermal (bremsstrahlung) absorption coefficient is given by (31), which determines the efficiency of the laser radiation absorption in laser thermonuclear fusion.

The laser radiation intensity, (W/cm^2), is denoted by I_0. The "overthreshold coefficient," m, determines the strength of the parametric coupling — the scaling of the parametric growth rate with the intensity of laser radiation electric field E_0.

Parametric Coupling	**Coupling Region Δx_T**	**Threshold value**	**Growth rate**	**ν_T**	**n**	**m (ν_T/ν_{ei}) $(\Delta x_T/L_N)$**
Two Plasmon Decay	**$\sim L_N \ln^{-1} (\omega_{pe}/\nu_{ei})$**	**"Absolute" See (17)**	**See (18)**	**See (19)**	**n>>1**	**High Parametric Absorption**
Stimulated Raman Scattering	**$\sim L_N \ln^{-1} (\omega_{pe}/\nu_{ei})$**	**"Absolute" See (24)**	**See (25)**	**See (28)**	**n>>1**	**High to Medium Parametric Absorption**
Plasmon - Phonon Decay	**$\sim L_N$**	**"Convective" See (20)**	**See (21)**	**See (22)**	**n>>1**	**Medium to Low**
Stimulated Brillouin Scattering	**$\sim L_N$**	**"Convective" See(29)**	**See (30)**	**Low**	**n>>1**	**Low**

Table1. Laser thermonuclear fusion: Efficiency (m) of the laser radiation absorption via parametric turbulence channels. Summary of the above discussed parametric effects. The estimates of "m" are based on the parameters in the experiments in the 1970s.

It is to be noted that in the 1970s, in some papers the interpretation of the experiments with the same I_0 led to confusion. As I_0 includes three parameters: laser radiation energy, laser pulse duration, and the laser focusing area, obviously the physics is not the same as these parameters are changed for the same I_0.

For the absolute parametric decay, t → l + s, and oscillating two stream instability, t → l + a, $(\Delta x_T / L_N) < < 1$, so that (31) is satisfied for large m. In contrast to the parametric decay instability, which can be either absolute or convective, the oscillating two stream instability is an absolute instability. For the oscillating two stream instability $(\Delta x_T / L_N) \leq 1$, and, accordingly, it is more favorable than the absolute parametric decay instability.

In the case of two plasmon decay in the quarter critical region, which can be either convective or absolute, $(\Delta x_T / L_N) <1$, so that (23) can be satisfied with relatively small m.

Saturation of Raman scattering instability via strong plasma turbulence process, (modulational instability), leads to the caviton formation, which propagate into the lower density regions. There the plasmon collapse occurs and the plasmon energy is transferred, through linear Landau damping, to electrons. This is one of the mechanisms, leading to generation of suprathermal electrons {reviewed by Stefan (1989) {17.b]}. Here $(\Delta x_T / L_N) <1$, so that, according to (31) and (32), a strong laser radiation absorption takes place for larger m. The summary of the effects discussed is given in the Table 1.

In the magnetic confinement thermonuclear fusion research, the parametric phenomena are richer due to the variety of magnetized plasma modes and possible primary and secondary parametric excitations.

7. A General Comment on the Parametric Plasma Turbulence

The picture of the parametric saturation processes, however, is far from being clear, as the saturation mechanisms swiftly interchange their importance, depending on the dynamics of the particular parametric interaction. In other words, parametric plasma turbulence leads to a "turbulent" system of possibilities, not easily identified by the theorists and verified by the experimentalists [30].

8. Marshall Rosenbluth: A Life in the Thermonuclear Fusion Research

Marshall Rosenbluth started his research in thermonuclear physics through his involvement in the development of the American hydrogen bomb {Teller (1982) [31]} He ended up his research as one of the ultimate authorities in thermonuclear fusion research and a passionate advocate for a peaceful use of thermonuclear energy.

As he was observing a 15 megatons thermonuclear bomb explosion in the South Pacific, in 1954, Rosenbluth said,[4]

"*The thing was glowing. …It was a much more awesome sight than a punny little atomic bomb. It was a pretty sobering and shattering experience*."

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[4] Freeman Dyson, Marshall N. Rosenbluth: Biographical Memoirs. Proceedings of the American Philosophical Society, Vol. 150, No. 2, June 2006.

Thereupon, Rosenbluth dedicated his life to the development of a peaceful thermonuclear fusion. His initial idea was to use a deuterium-tritium pinch plasma confined by a magnetic field.[5] Many of his papers became widely known… on stabilization of flute perturbations by finite Larmor radius (with N. A. Krall and N. Rostoker)… on the trapped particle instability (with H. Berk).[6]

The "burning of fusion plasma" has been a dream of every plasma thermonuclear fusion scientist. In 2000, about the "yearning and burning" Rosenbluth said,[7]

"As we have realized for many years, the point at which science and the fusion energy goal converge is in a burning plasma experiment… We can imagine many possible experiments, covering a wide range of goals and costs…Applicability of results from a Tokamak burning experiment to other concepts… where other phenomena may be dominant is still more speculative. It is likely that we are many, many years away from being able to consider seriously a non-Tokamak burning plasma experiment… Let's move expeditiously from yearning to learning."

References and Notes

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[5] V. D. Shafranov, In Memeory of Marshall Nicholas Rosenbluth (February 5, 1927 – September 28, 2003), Plasma Physics Report, Vol. 30, No.3, 2004, pp.274-275. Translated from Fizika Plazmy, Vol.30, No.3, 2004, pp.303-304.

[6] E. P. Velikhov, M. N. Rosenbluth: Statement from the Kurchatov Institute (Moscow, 2003).

[7] Marshall Rosenbluth, From Yearning to Burning: Possible Broad-brush Guidelines for "Burning Plasma" Thinking: the paper of December 6, 2000, (General Atomic, San Diego, California).

1. M. N. Rosenbluth, Parametric Instabilities in Inhomogeneous Media, *Phys. Rev. Letters*, **29**, 565 (1972).

This is the seed paper, which has initiated a vigorous research in the convective-absolute parametric instabilities in the laboratories worldwide.

2. M. N. Rosenbluth and R. Z. Sagdeev, Laser Fusion and Parametric Instabilities, Comments Plasma Phys. Thermonucl. Res., **1**, 129 (1973).

This paper gives a comprehensive and simple, but not elementary, physical picture of the parametric instabilities in a laser thermonuclear fusion research, including both linear and nonlinear theory of parametric instabilities. Linear stabilization processes of the parametric instabilities: collisional and Landau damping; finite bandwidth of the laser radiation; plasma inhomogeneity. Nonlinear saturation processes: the Langmuir mode cascading, (weak parametric turbulence), and Langmuir mode modulational instability, (strong parametric turbulence). The concept of anomalous collision frequency ν_a is used to describe the dissipation of laser radiation energy.

3. A. A. Galleev, G. Laval, T. M. O'Neil, M. Rosenbluth, and R. Z. Sagdeev, JETP, **65**, 973 (1973).

4. The character of the parametric instabilities, convective versus absolute, is of utmost importance in laser thermonuclear fusion research. In the case of convective parametric instabilities, the saturation in the region of parametric excitation is achieved through the balance of the convected and absorbed energies. Both processes are linear with respect to the plasmon amplitude. The parametric excitation, however, is nonlinear in reference to the laser electric field amplitude. The convective parametric instabilities, in general, lead to a low laser energy deposition. Accordingly, the convective parametric instabilities are not favorable from the standpoint of laser radiation deposition in plasma corona.

In the case of absolute instabilities, the excited plasmon is not convected away, but is "trapped" in the parametric excitation region. It grows until saturation is reached, resulting in a stationary turbulent plasma state. This, as a rule, leads to a high level of laser energy deposition. That’s why the absolute instabilities are more favorable for the laser fusion.

5. K. A. Brueckner and S. Jorna (1974), *Laser Driven Fusion*, Rev. Mod. Physics, **46**, 325, (1974).

This work is also available in Russian: *Lazerniy Syntez,* Nauka, Moscow, (1978).

6. Yu. V. Afanasiev, N. G. Basov, O. N. Krokhin, V. V. Pustovalov, V. P. Silin, G. V. Sklizkov, V. T. Tikhonchuk, A. S. Shikanov, *Interaction of a Strong Laser Radiation with Plasma,* Radiotekhnika, Vol. 17, Moskow (1978), in Russian.[Izvestiya Vysshikh Uchebnykh Zavedenii, *Radiofizika,* **17**, *Moskva* 1978].

In this book, a thorough presentation of the research worldwide in plasma parametric turbulence in laser fusion research in the 1970s is given, pp.101-162.

7. John Nuckolls, Lowell Wood, Albert Thiessen, and George Zimmerman, Laser Compression of Matter to Super-High Densities: Thermonuclear (CTR) Applications, *Nature* **239**, 139 (September 15, 1972).

8. R. Kidder (1991); A presentation by R. Kidder at the conference “Achievements in Physics,” honoring Keith A. Brueckner, in late January 1991. Kidder talked about history of the development of the laser fusion program at Lawrence Livermore National laboratory.

9. K. A. Brueckner, Editor-in-Chief, *Inertial Confinement Fusion* (American Institute of Physics, New York, (1992); Research Trends in Physics Series founded and edited by V. Alexander Stefan).

The Conference was held in La Valencia Hotel, La Jolla, California, February 4-6, 1991, and was hosted by La Jolla International School of Physics.

Marshall Rosenbluth attended this conference. {See the photo of Marshall Rosenbluth in conversation with Keith Brueckner}.

January 1991, La Jolla, California.

Keith A. Brueckner, (b. 1924), the founder of the Physics Department at the University of California, La Jolla, and **Marshall N. Rosenbluth** (1927—2003), the founder of theInstitute for Fusion Studies in Austin, Texas.

10. M.N. Rosenbluth, Editor-in-Chief, *New Ideas in Tokamak Confinement* (American Institute of Physics, New York, (1992); Research Trends in Physics Series founded and edited by V. Alexander Stefan).

The Conference was held in La Valencia Hotel, La Jolla, California, January 27-29, 1992, and was hosted by La Jolla International School of Physics.

Rosenbluth attended the conference. Rostoker, Krall, and Berk, the contributors to the present volume, also attended the conference {See Part IV: The Photo Gallery}.

11. The paper initiated a number of research programs worldwide concerning the character of parametric instabilities, convective versus absolute, in connection with the controlled thermonuclear fusion research. I know first hand that the Rosenbluth paper was of a high concern for the laser fusion program at the Lebedev Institute of the

Russian Academy of Sciences led by Nikolay Genadievich Basov,[2] who, with Oleg N**.** Krokhin**,**[3] in the early 1960s in Russia (then the Soviet Union)[N.G Basov and O.N. Krokhin, Zh. Eksp. Teor. Fiz. 46, 171, (1964) [Sov. Phys. JETP 19,123, (1964)]. and John M. Dawson[4] in the States [J.M Dawson,., Phys. Fluids, 7, 981, (1964)], independently, proposed lasers in interaction with the fusion pellets as a new thermonuclear fusion scheme. I was in Moscow, on and off, from late 1977 to mid-1981, and was a participant in the research on that subject in the Lebedev (Department of the Plasma Theory founded and led by Victor Pavlovich Silin[5]). Similar work on laser-plasma interactions and Tokamak confined plasmas was done at the MIT Plasma Fusion Center led by Abraham Bers,[6] who had extensively studied the problem earlier. For the Elmo Bumpy Torus and magnetic mirror plasmas, the convective-absolute character of parametric instabilities was addressed at JAYCOR, Inc. led by Nicholas A. Krall.[7]

12. E. Teller, The Work of Many People, Science **121**, 267 (February 25, 1955).

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[2] N. G. Basov (1922—2001), a Russian physicist; winner of a 1964 Nobel Prize in physics with A. M. Prokhorov and C. H. Townes

[3] O. N. Krokhin (b.1927), a Russian physicist, known for his research in laser fusion.

[4] J.M. Dawson (1930—2001), an American physicist, known for his research in plasma particle accelerators and computational plasma physics.

[5] V. P. Silin (b.1927), a Russian physicist, known for his work in nonlinear plasma theory and superconductivity.

[6] A. Bers (b.1930), a Romanian born American physicist, known for his works on the space-time evolution of instabilities, RF current drive in Tokamaks, and nonlinear plasma physics.

[7] N. A. Krall (b.1932), an American physicist, known for his classic textbook: Principles of Plasma Physics, he co-authored with A. Trivelpiece, and his research in alternative concepts in controlled thermonuclear fusion.

13. **a**). A. D. Piliya, JETP Letters, **17**, 374 (1973);

b). A. D. Piliya, Zh. Eksp. Teor. Fiz. 64, 1237 (1973) [Sov. Phys. JETP 37, 629 (1973)].

14 F. W. Perkins and J. Flick, Phys. Parametric Instabilities in Inhomogeneous Plasma, Fluids, **14**, 2012 (1971).

In the paper it is shown that the threshold value of laser radiation electric field for both, the aperiodic "oscillating two-stream" instability and the periodic decay instabilities, increases when the density-gradient scale length L_N becomes less than the electron mean free path l_{ei}.

15. a). V. P. Silin and A. N. Starodub, Absolute Parametric Instability of Inhomogeneous Plasma Zh. Eksp. Teor. Fiz., **66** 176 (1974); Sov. Phys. JETP, **39**, 82 (1974).

In this paper both plasma and laser radiation electric field is considered to be inhomogeneous. It is shown that two plasmon instability has an absolute character.

b). V. P. Silin and A. N. Starodub, Parametric Decay in a Nonuniform Plasma, Fizika Plazmi, **3**, 280 (1977), in Russian.

In the paper the absolute instability associated with the parametric conversion of the laser radiation into plasmon, (electron Langmuir mode), and phonon, (ion acoustic wave), is studied. The consequences for the laser thermonuclear fusion are discussed.

c). V. P. Silin and A.N. Starodub, Stimulated Raman Scattering in an Inhomogeneous plasma Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki, 67, 2110 (1974), in Russian.

It is shown that stimulated Raman scattering is an absolute parameteric instability. The laser radiation intensity threshold value is calculated.

16. V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasmas*, [Nauka Moscow, (1964), in Russian]; Pergamon, New York, (1970).

17. a). V. Stefan, N. A. Krall, J. B. McBride The Nonlinear Eikonal Relation of a Weakly Inhomogeneous Magnetized Plasma Upon the Action of Arbitrarily Polarized Finite Wavelength Electromagnetic Waves, Physics of Fluids, **30**, 3703 (1987).

In this paper the nonlinear eikonal relation of a weakly inhomogeneous magnetized plasma upon the action of arbitrarily polarized finite wavelength electromagnetic waves is studied. The plasma perturbations are treated within the framework of weak turbulence theory. The absorption and scattering parametric instabilities are treated as an eigen value problem, leading to a variety of dispersion relations. In [1], Rosenbluth treated the two plasmon parametric coupling as an eigen function problem.

b). V. Stefan. *Laser Fusion Research: Generation of Suprathermal Particles, Laser Radiation Harmonics, and d.c. Magnetic Felds* (Bulletin of the Stefan University, **Vol.1**, No.1, (1989); This book is available as a microfiche at the NTIS: National Technical Information Service, Washington, D.C.; NTIS No.: PB 94131463, 165 pages).

c). V. Stefan, B. I. Cohen, and C. Joshi, Nonlinear Mixing of Electromagnetic Waves in Plasmas, *Science*, **243**. no. 4890, pp. 494 – 500 (27 January 1989).

A seminal work of Rosenbluth in the beat wave plasma interaction has initiated a vigorous research worldwide in laser and Tokamak thermonuclear fusion research. In this paper, some aspects of that research are reviewed; focusing on plasma particle acceleration, free electron laser, and r.f. current drive in Tokamaks.

18. N. A. Krall and A. Trivelpiece, *The Priciples of Plasma Physics* (McGraw-Hill, New York, 1973).

19. The counter propagation of the parametrically excited modes is a necessary condition for the absolute instability. The modes propagating in the same direction are convectively unstable. The sufficient condition for the evaluation of the instability character was studied by: A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N.

Stepanov, *Plasma Electrodynamics* [Nauka, Moscow (1974), in Russian].

20. Y. C. Lee and P. K. Kaw, Temporal Electrostatic Instabilities in Inhomogeneous Plasmas, Phys. Rev. Lett., **32**, 135 (1974).

It is shown that two plasmon parametric instability becomes absolute due to the wave trapping, which prevents the convection of wave energy out of the unstable region.

21. C. S. Liu and M. N. Rosenbluth, Parametric Decay of Electromagnetic Waves into Two Plasmons and Its Consequences, Phys. Fluids, **19**, 967 (1976).

The growth rate and threshold of the absolute two plasmon instability are derived. The bound on the saturation level of the plasma waves is obtained from the consideration of the pump depletion. The scattered radiation at $3\omega_0/2$ due to the beating of the incident wave with the backward plasma wave is calculated and compared with observations.

22. M.N.Rosenbluth, R.B.White, and C.S.Liu, Temporal Evolution of a Three-Wave Parametric Instability, Physical Review Letters, 31, 1190 (1973).

A three-wave parametric instability in an inhomogeneous plasma is studied. An initial fluctuation develops into a pulse and grows initially with the same growth rate as it would in a homogeneous plasma.

23. C. S. Liu, M. N. Rosenbluth, and R. B. White, Parametric Scattering Instabilities in Inhomogeneous Plasmas, Phys. Rev. Lett. **31**, 697 (1973).

Raman and Brillouin scatterings in inhomogeneous and blowoff plasmas are studied. It is shown that for normal incidence, the threshold for near-90° Raman scattering is a minimum, necessitating the analysis without the WKB approximation.

24. a. V. P. Silin, Parametric *Action of High Power Radiation on Plasmas* (Nauka, Moscow, 1973).

b. V. P. Silin and V. T. Tikhoncuk**,** Physics Reports (1985).

25.a. P. K. Kaw, W. L. Kruer, C. S. Liu, and K. Nishikawa , in *Advances in Plasma Physics*, edited by A. Simon and W. B. Thompson (Interscience, New York, 1976), Volume 6.

b. C. S. Liu and V. K. Tripathi, Interaction of Electromagnetic Waves with Elecron Beams and Plasmas (World Scientific (1994).

c. W. L. Kruer, The Physics of Laser Plasma Interactions (Westwood Press (2003).

26. A. A. Galeev, V. N. Oraevsky, and R. Z. Sagdeev, JETP Letters, **16**, 194 (1972), in Russian.

27. A. Bruce Langdon, Barbara F. Lasinski, and William L. Kruer, Nonlinear Saturation and Recurrence of the Two-Plasmon Decay Instability, Phys. Rev. Lett., **43**, 133 (1979).

In this paper, a 2D computer simulation is carried out, showing that short-wavelength ion fluctuations and density profile steepening are dominant factors in the saturation of the two plasmon instability.

28. The saturation process will unfold through the channel, (through the mechanism), that leads to the lowest turbulence level. The system of the saturation mechanisms can be compared to the electrical system of impedances in a parallel coupling. The maximum current, (the most favorable saturation process), flows through the brunch, (the channel of saturation), with the lowest impedance, (the lowest turbulence level).

29. V.E Zakharov, JETP, **62**, 1745 (1972)

Modulational instability and the collapse of electron plasma modes is studied.

30. At the turn of the 21st, plasma parametric turbulence is still a problem for overall fusion program, both inertial and magnetic confinement fusion. The wisdom lingers upon a fusion physicist: avoid dealing with plasma parametric turbulence whenever possible. The legacy of Werner Heisenberg[8] is present in the community of the

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[8] Werner Heisenberg (1901—1976), a German physicist.

thermonuclear fusion physicists. Heisenberg had almost flunked on his Doctoral Dissertation exam, in 1923, had it not been for Arnold Sommerfeld,[9] his mentor. When asked by Wien[10] about the experimental consequences of his work on the hydrodynamic turbulence, Heisenberg could not give a satisfying answer to the Examining Board. Sommerfeld passionately defended Heisenberg by saying that a theoretical physicist did not have to know the details of experiments.

The schism between the theoretical and experimental physicists has not faded away. This is the same Sommerfeld who advised his students, Heisenberg and Pauli[11] among others, to continue with the calculation in quantum mechanics, hoping that the true meaning of the calculation would be revealed along the way. Sommerfeld hope has failed.

Quantum mechanics of today, as was of yester days, is abundant with different interpretations, as is the parametric plasma turbulence, and turbulence in general. Lev Landau,[12] known in plasma physics, among other achievements, for his Landau damping, which was first experimentally verified by Malmberg[13] of University of California in La Jolla, CA, used to say that the turbulence is the most difficult among all the problems in physics.

31. Edward Teller, Hydrogen Bomb History, *Science*, **218**, No. 4579, 1270 (December 24, 1982).

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[9] Arnold Sommerfeld (1868—1951), a German physicist.
[10] Wilhelm Wien (1864—1928), a German physicist.
[11] Wolfgang Pauli (1900—1958), an Austrian physicist.
[12] Lev Landau (1908—1968), a Soviet physicist.
[13] John H. Malmberg (1927—1992), an American physicist.